SUPERSYMMETRIC FIELD THEORY WITH BENIGN GHOSTS

Nantes, 18 décembre 2013

based on
A.S., arXiv:1306.6066 [hep-th]

MOTIVATION:

Problems with causality in quantum and classical gravity.

Dream solution:

[A.S., 2005]

• our Universe as a soap film in a flat higher dimensional bulk. The TOE is a field theory in this bulk.

• To be renormalizable, it should involve higher derivatives

DANGER: the ghosts

• GHOSTS = instability (rather *absence*) of vacuum

• inherent for higher-derivative theories.

Conventional system

$$E = \frac{\dot{q}^2}{2} + V(q)$$

can have a classical and/or quantum bottom

• Consider Pais-Uhlenbeck oscillator.

$$L = \frac{1}{2}(\ddot{q} + \Omega^2 q)^2 \ .$$

Then

$$E = \ddot{q}(\ddot{q} + \Omega^2 q) - \dot{q}(q^{(3)} + \Omega^2 \dot{q}) - \frac{1}{2}(\ddot{q} + \Omega^2 q)^2$$

can be as negative as one wishes.

- Common lore: negative residues in propagators break unitarity.

NOT TRUE!!

- No problem in free theories

- Interactions may lead to collapse and breaking of unitarity. Like falling into the center in the attractive potential $\sim 1/r^2$.

- If quantum theory is sick, so is its classical counterpart. If classical theory is benign, so is its quantum counterpart

Benign nonlinear SQM system with ghosts
[D. Robert + A.S., 2006]

$$S = \int dt dx d\bar{\theta} d\theta \left[\frac{i}{2} \bar{\mathcal{D}}\Phi \frac{d}{dt} \mathcal{D}\Phi + V(\Phi) \right] ,$$

with the real (0+1)-dimensional superfield

$$\Phi = \phi + \theta\bar{\psi} + \psi\bar{\theta} + D\theta\bar{\theta}$$

- An extra time derivative.

The Hamiltonian

$$H = pP - DV'(\phi) + \text{fermion term}$$

is not positive definite.

Integrals of motion:

1. $E \equiv H$
2. $N = \frac{1}{2}\dot{\phi}^2 - V(\phi)$

- Exactly solvable.
- Take

$$V(\Phi) = -\frac{\omega^2 \Phi^2}{2} - \frac{\lambda \Phi^4}{4},$$

- Explicit solutions

$$\phi(t) \;=\; \phi_0\,\mathrm{cn}[\Omega t|m]$$

with

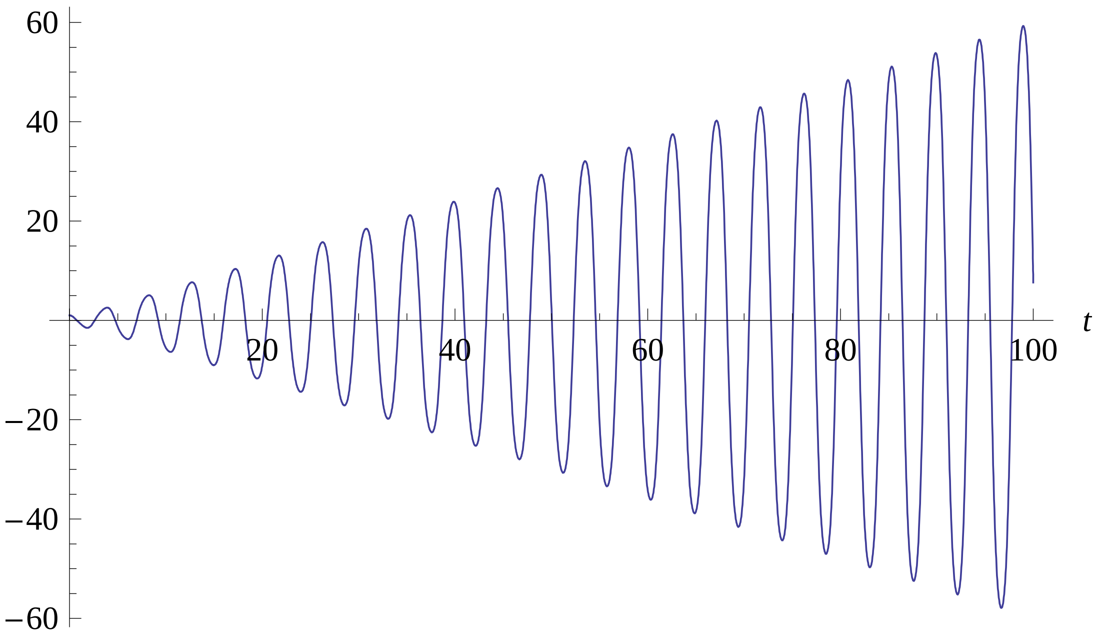
$$\alpha = \frac{\omega^4}{\lambda N}, \quad \Omega = [\lambda N(4+\alpha)]^{1/4}, \quad m = \frac{1}{2}\left[1 - \sqrt{\frac{\alpha}{4+\alpha}}\right],$$

$$\phi_0 \;=\; \left(\frac{N}{\lambda}\right)^{1/4}\sqrt{\sqrt{4+\alpha}-\sqrt{\alpha}}$$

$$D(t) \propto \dot{\phi}(t)\int^t \frac{dt'}{\dot{\phi}^2(t')}$$

- $\phi(t)$ is bounded.
- $D(t)$ grows linearly.

$D(t)$



• Quantum problem is also exactly solvable. Continuous spectrum, $E \in [-\infty, -\omega] \cup [\omega, \infty]$ + infinitely many normalizaed states with $E = 0$.

• Explicit expressions for wave functions exist.

• Evolution operator is unitary.

(1+1) field theory.

•. Let Φ depend on t and x. Choose

$$S \;=\; \int dt dx d\bar{\theta} d\theta \left[-2i\mathcal{D}\Phi\partial_+\mathcal{D}\Phi + V(\Phi)\right] ,$$

where $\partial_\pm = (\partial_t \pm \partial_x)/2$ and

$$\mathcal{D} = \frac{\partial}{\partial\theta} + i\theta\partial_-, \qquad\qquad \bar{\mathcal{D}} = \frac{\partial}{\partial\bar{\theta}} - i\bar{\theta}\partial_+$$

Bosonic Lagrangian

$$\mathcal{L}_B \;=\; \partial_\mu\phi\partial_\mu D + DV'(\phi)$$

Equations of motion

$$\begin{aligned} \Box\phi + \omega^2\phi + \lambda\phi^3 &= 0 \\ \Box D + D(\omega^2 + 3\lambda\phi^2) &= 0\,. \end{aligned}$$

Two integrals of motion:

$$E = \int dx \left[\dot{\phi}\dot{D} + \phi' D' + D\phi(\omega^2 + \lambda\phi^2)\right]$$

(positive or negative)

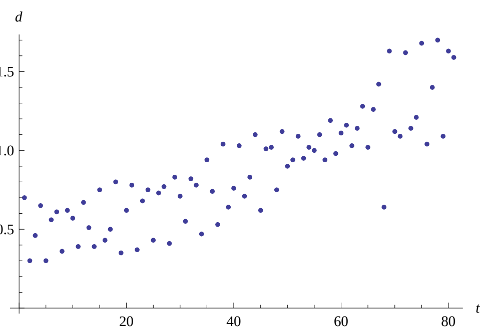
$$N = \int dx \left[\frac{1}{2}\left(\dot{\phi}^2 + (\phi')^2\right) + \frac{\omega^2\phi^2}{2} + \frac{\lambda\phi^4}{4}\right]$$

(positive definite)

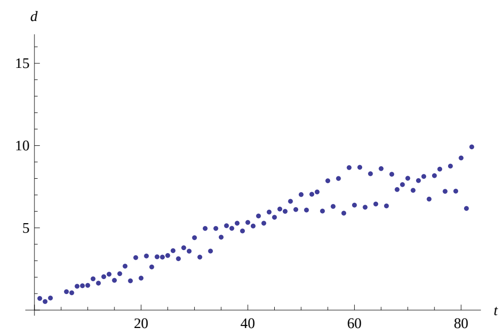
- Stochasticity. Solved numerically.

Initial conditions

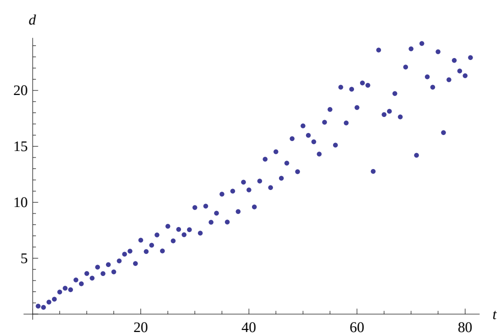
$$\phi(x,t) = Ce^{-x^2}, \quad \text{with} \quad C = 1, 3, 5$$
$$D(x,t) = \cos\pi x/L, \qquad L = 10 \ \text{being the length of the box}$$



(a) $C = 1$



(b) $C = 3$



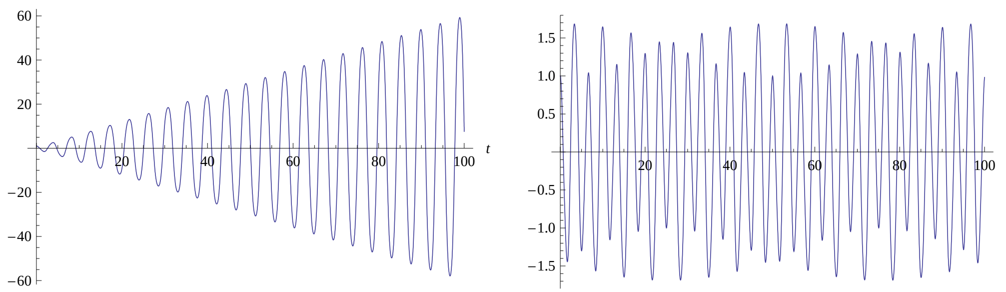
(c) $C = 5$

Figure 1: Dispersion $d = \sqrt{\langle D^2 \rangle_x}$ as a function of time for different values of C.

IMPLICATIONS FOR INFLATION ?

- Homogeneous classical field needed.
- Let $\phi(t)$ be homogeneous. Then different Fourier modes of $D(x,t)$ decouple.

ONLY ZERO MODE GROWS !



(a) $k = 0$ (b) $k = 1$

- $D(x,t)$ becomes more and more homogeneous.