Description théorique du plasma quark-gluon

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Description(s) théorique(s) du plasma quark-gluon et modèles choisis

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Plan

I. Introduction and motivation

II. QGP from the high T / weak coupling limit

III. Finite-T Lattice QCD & the phase transition

IV. pQCD at finite T: the Hard Thermal Loop approach

V. Effective models and URHIC

VI. Conclusions and Perspectives

Basics of QCD

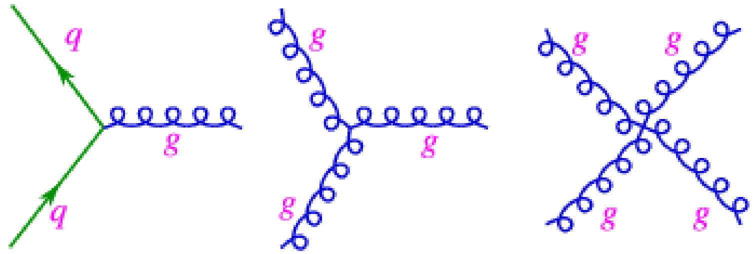
From J. Wambach (The Phase Diagram of Strongly Interacting Matter); 2006

$SU_c(3)$ YM theory as a model of strong interaction

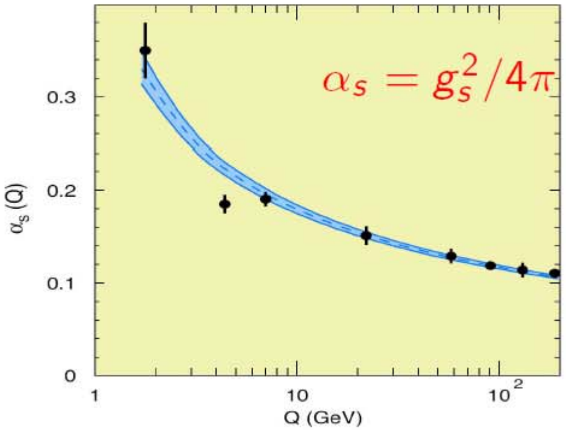
Nambu 1965

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + \bar{q}\gamma^\mu (i\partial_\mu - g_s \Lambda_a A^a_\mu) q - m_q \bar{q} q$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f_{abc} A^b_\mu A^c_\nu$$



Gross & Wilczek, Politzer 1973



asymptotic freedom $(Q \gg \Lambda_{QCD})$

confinement $(Q \sim \Lambda_{QCD})$

spontaneous chiral symmetry breaking

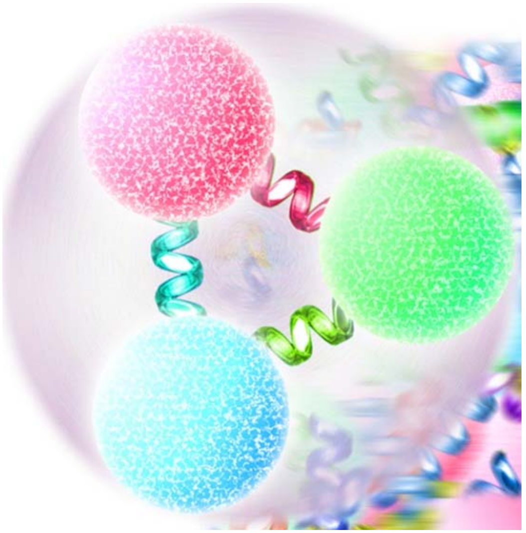
in the physical vacuum quarks and gluons condense

$$\rightarrow \quad \langle \bar{q}q \rangle \sim (240 \text{ MeV})^3; \quad \langle (gG^a_{\mu\nu})^2 \rangle \equiv \langle G^2 \rangle \sim (850 \text{ MeV})^4$$

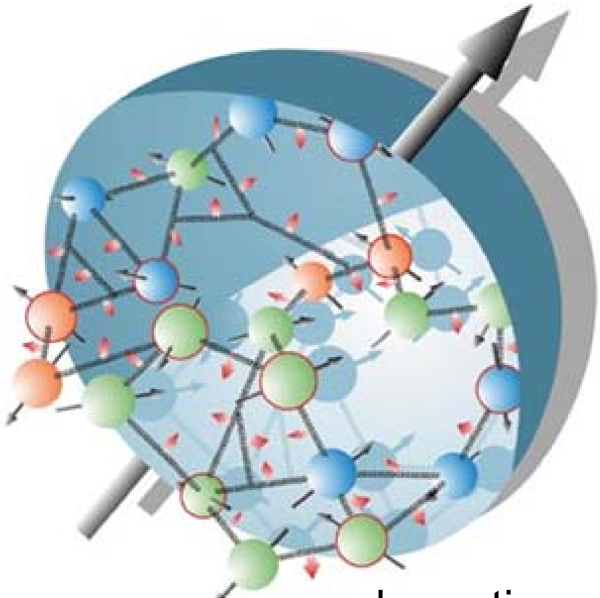
huge numbers! $\langle \bar{q}q \rangle \sim 1.8 \frac{\text{pairs}}{\text{fm}^3}; \quad \langle G^2 \rangle \sim 70 \frac{M_N}{\text{fm}^3}$

Specific aspects of QCD (wrt usual nucl. Φ)

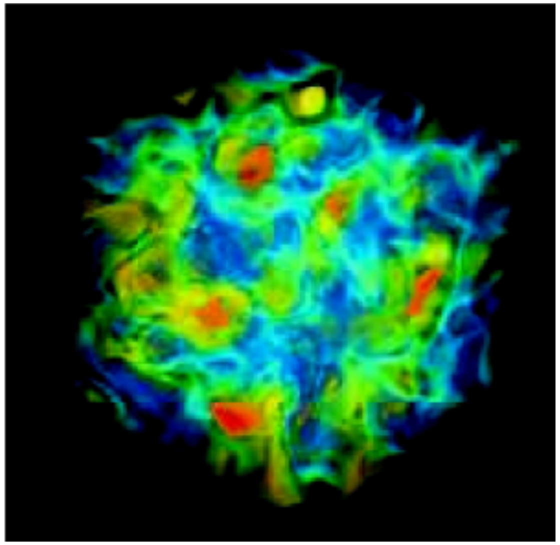
Binding E>>mc^2	Binding E<<mc^2
∞-body problem	n-body problem
"Simple" forces btwn quarks	"Complicated" forces btwn nucleons
Complicated agregates and effective dof (Q^2 dependent)	Agregates of moderate complexity



pictorial

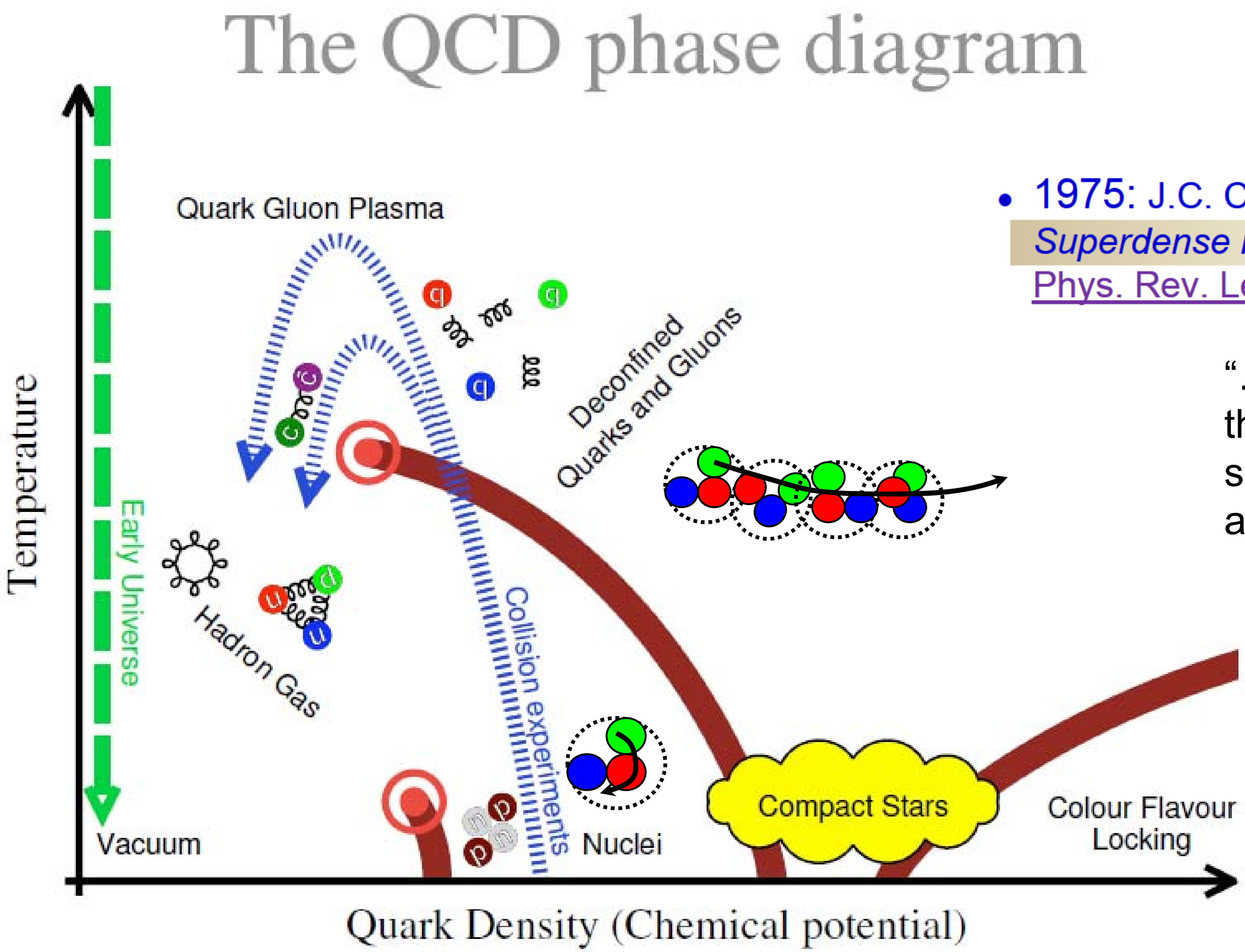


schematic



realistic

QCD matter under extreme conditions

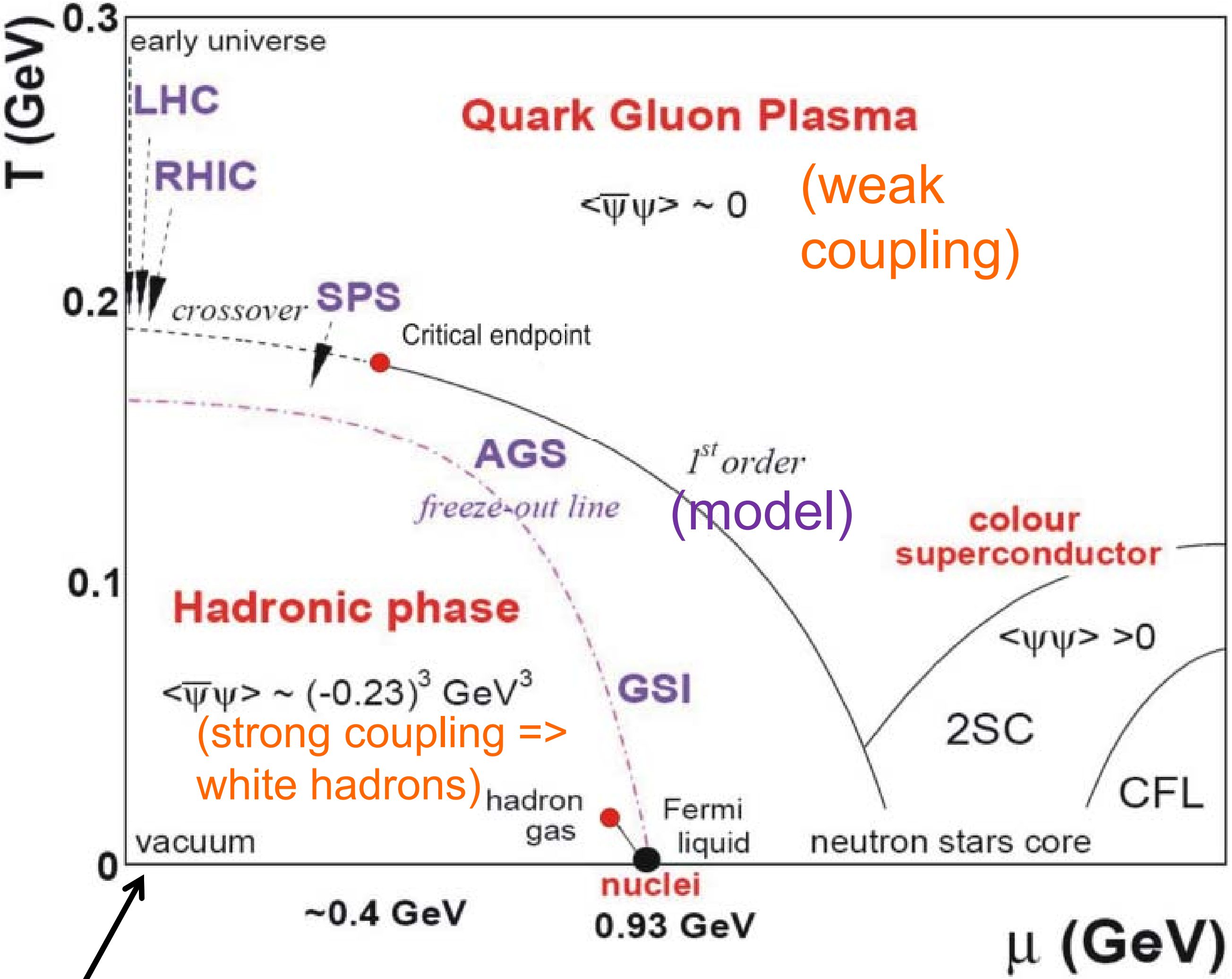


- **1975:** J.C. Collins and M.J. Perry, *Superdense Matter or Asymptotically Free Quarks?*, Phys. Rev. Lett. **34**, 1353.

"…matter at densities higher than nuclear consists of a quark soup. The quarks become free at sufficiently high density."

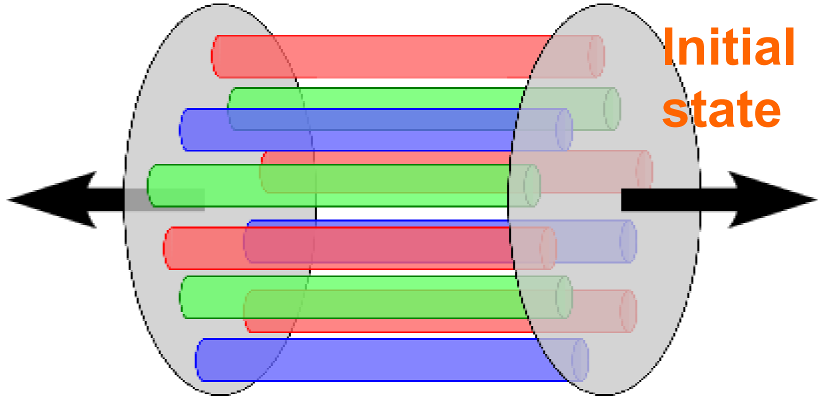
Naive view: larger temperature T (or larger baryonic density ρ_B) $\Rightarrow$ larger hadronic density $\Rightarrow$ overlapping of individual hadrons $\Rightarrow$ possible *tunneling* of single quarks:

Matter under extreme conditions

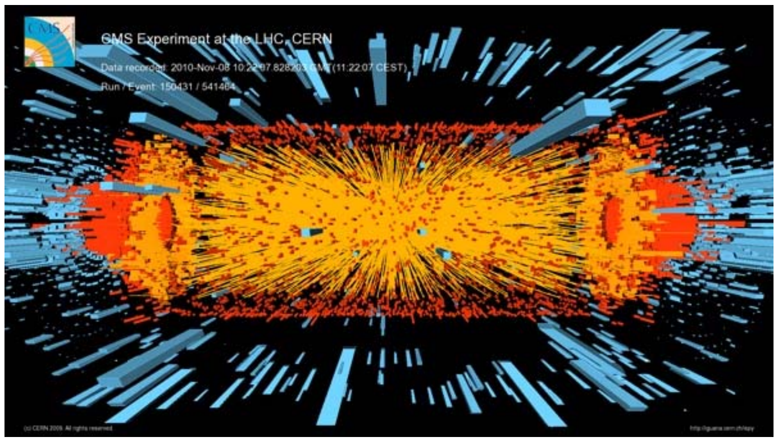


From J. Wambach (The Phase Diagram of Strongly Interacting Matter); 2006

Smaller μ at larger collider energies ?



Initial state



Final state

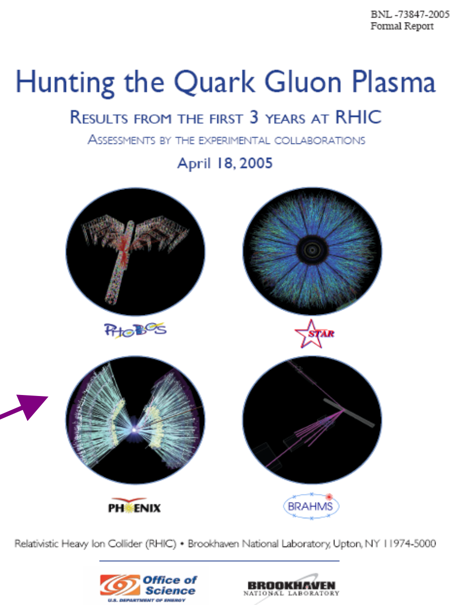
Baryon poor

Baryon rich

Investigating the Quark Gluon Plasma, why ?

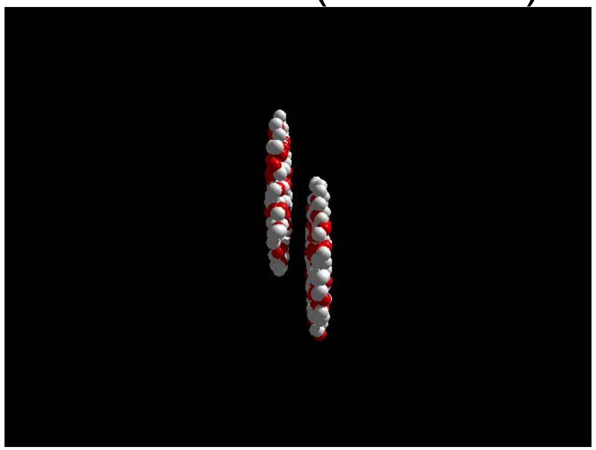
Possible interests (intrinsic & extrinsic) of QGP study:

- One of the strongest coupled many-body system (new techniques, new concepts) ⇒ Challenging per se
- Could help in understanding *some aspects* of confinement
- Ingredient of the astrophysical "standard model"
- It has probably been (re)created in earth during the last decade thanks to URHIC: it *EXISTS* and should be characterized!

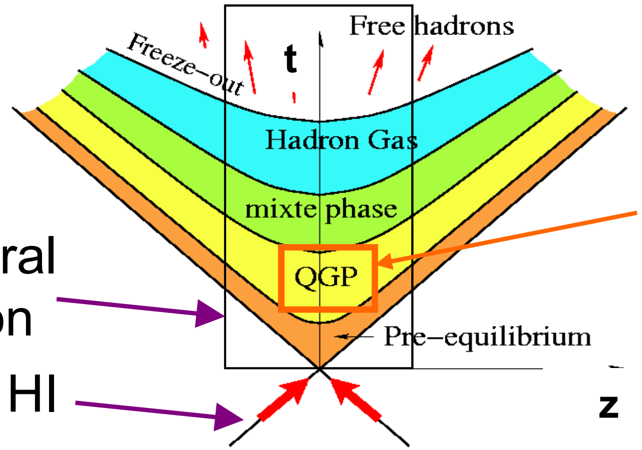


Ultra-Relativistic Heavy Ion Collisions

Schematic view I (URQMD):



Schematic view II (time – long. direction)



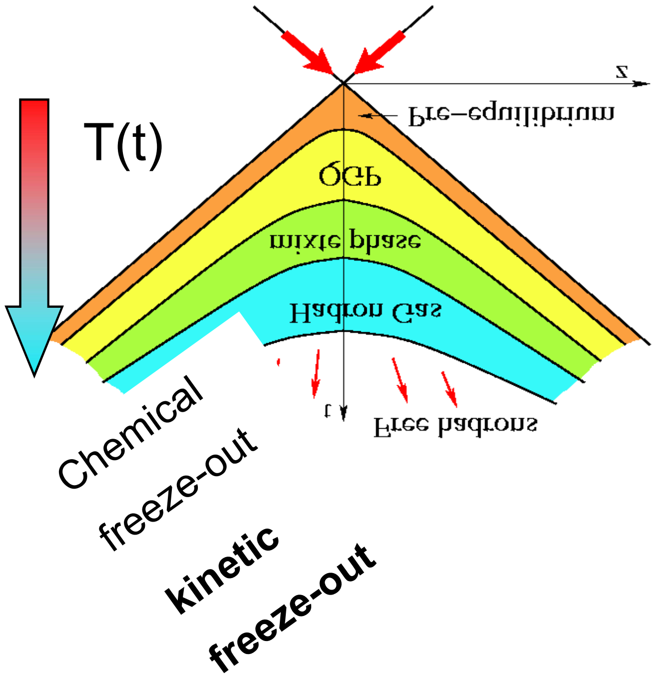
Since mid-80's → now (AGS, SPS, RHIC, LHC): more and more energy deposit in the central overlapping region.

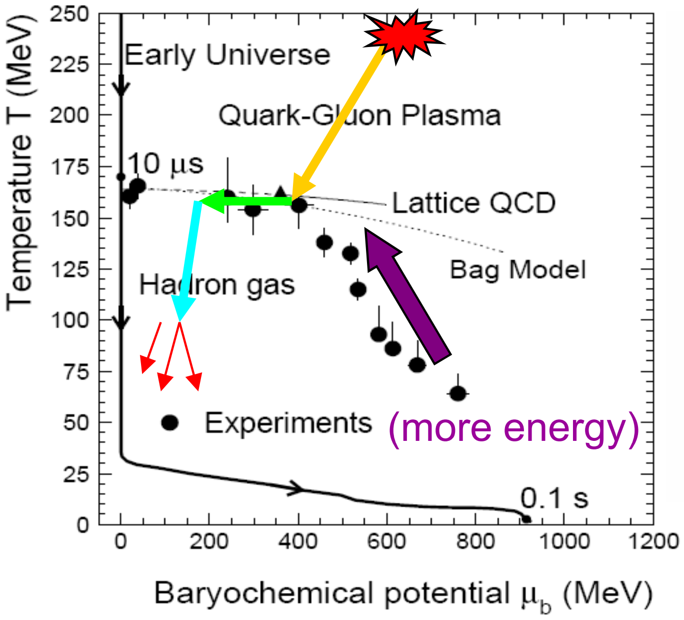
One system, many questions

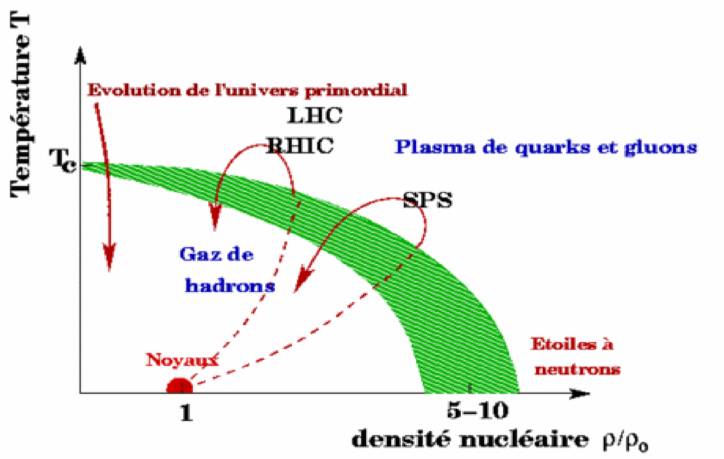
I. Does the system created in central region reach and maintain equilibrium long enough to be understood in terms of a quasi-stationnary state ?

Hadro-chemistry as a *thermometer* (# and spectra):

P. Braun-Munzinger & J. Wambach (arXiv:0801.4256)







One recovers the naïve view

Experiments seem to reveal the freeze-out horizon, i.e. the frontier between a hadron gas and a state “beyond”

QGP at large T (naïve pQGP)

Naïve idea (80's-90's): $\alpha_s(T >> \Lambda_{QCD}) << 1$ => gas of non interacting partons (pot. Energy ≈ $\alpha_s(T)$ << T: kin. Energy) => SB law

Partition function of quantum system:

$$Z = \sum_n \langle n | e^{-(\hat{H}-\mu\hat{N})/T} | n \rangle$$

For bosons with *d* internal dof (μ=0):

$$Z_B = \Pi_k \left(1 - e^{-E(k)/T}\right)^{-d}$$

$$p = T \left.\frac{\partial \ln Z_B}{\partial V}\right|_{T,\mu} = -d \int \frac{d^3k}{(2\pi)^3} T \ln\left(1 - e^{-E(k)/T}\right) \to d\frac{\pi^2}{90}T^4$$

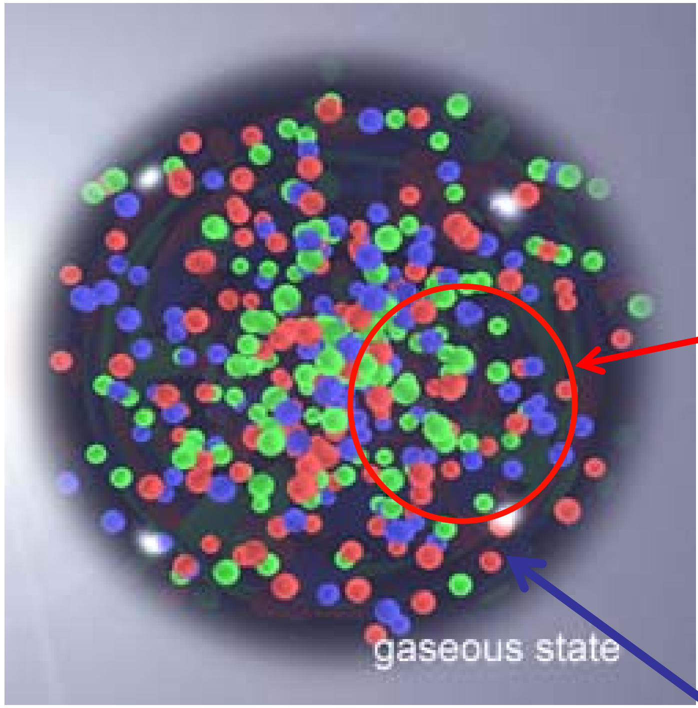
When T>>m

For fermions with *d* internal dof (μ=0):

$$Z_F = \Pi_k \left(1 + e^{-(E(k)-\mu)/T}\right)^{d}$$

$$p = T \left.\frac{\partial \ln Z_F}{\partial V}\right|_{T,\mu} = d \int \frac{d^3k}{(2\pi)^3} T \ln\left(1 + e^{-E(k)/T}\right) \to d\frac{7}{8}\frac{\pi^2}{90}T^4$$

Naïve pQGP



$$\begin{cases} p = \underbrace{\left(d_g + \frac{7}{8} d_q\right)}_{d_{QGP}} \frac{\pi^2}{90} T^4 \\ \epsilon = 3p = 3 d_{QGP} \frac{\pi^2}{90} T^4 \\ s = \frac{p+\epsilon}{T} = 4 d_{QGP} \frac{\pi^2}{90} T^3 \end{cases}$$

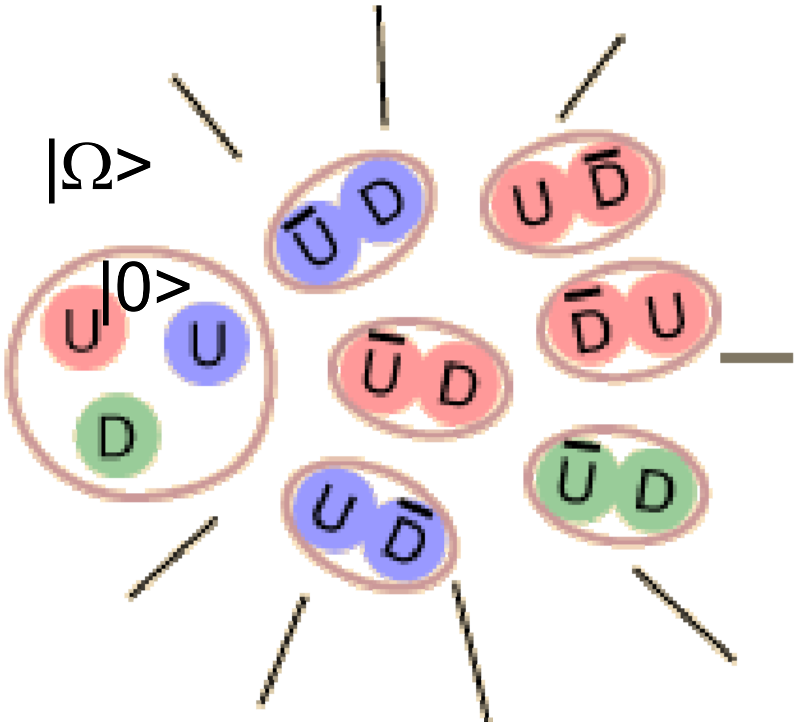
with d_{QGP}=37 for N_f=2

On the top of some "perturbative" vacuum |0>

(no condensates)

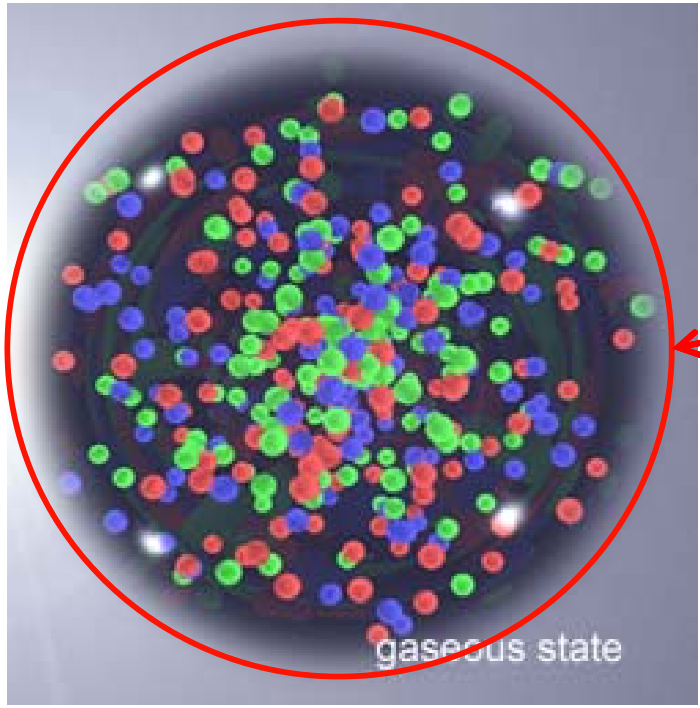
Naïve pQGP

MIT Bag model of hadrons:



Pressure from $|\Omega\rangle \rightarrow |0\rangle$: $B \approx (220\ \text{MeV})^4$.

Naïve pQGP



$$\begin{cases} p_{QGP} = d_{QGP}\dfrac{\pi^2}{90}T^4 - B \\ \epsilon_{QGP} = 3d_{QGP}\dfrac{\pi^2}{90}T^4 + B \\ s_{QGP} = \dfrac{p_{QGP}+\epsilon_{QGP}}{T} = 4d_{QGP}\dfrac{\pi^2}{90}T^3 \end{cases}$$

with d_{QGP}=37 for N_f=2

To be compared with a pion gas:

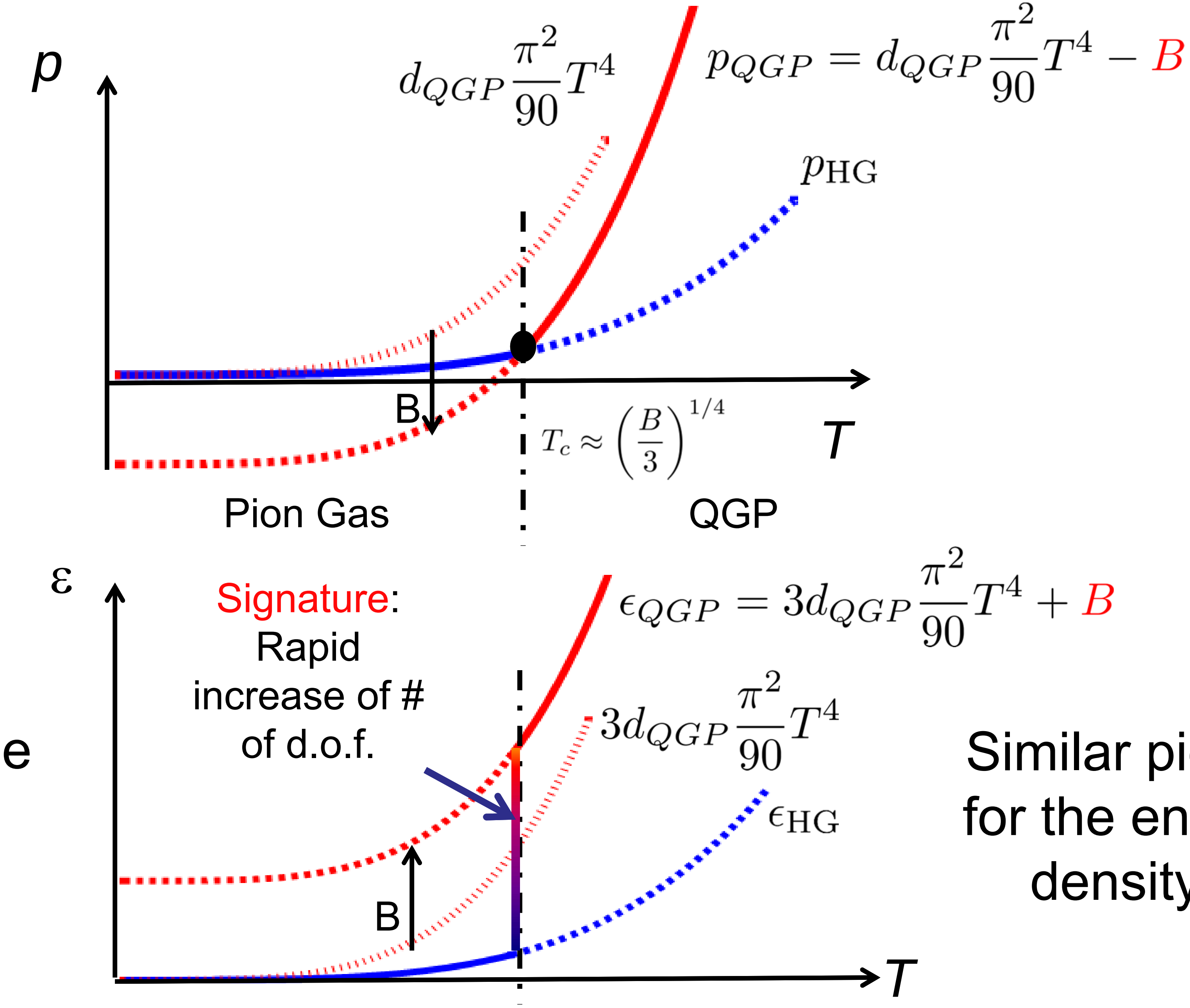
$$\begin{cases} p_{\mathrm{HG}} = d_\pi\dfrac{\pi^2}{90}T^4 \\ \epsilon_{\mathrm{HG}} = 3d_\pi\dfrac{\pi^2}{90}T^4 \\ s_{\mathrm{HG}} = 4d_\pi\dfrac{\pi^2}{90}T^3 \end{cases}$$

with d_π=3

Naïve pQGP

Equilibrium condition:

1rst order phase transition



Similar picture for the entropy density *s*

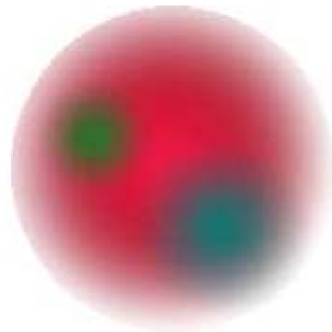
Rigorous formulation: QCD on the lattice

From Christian B. Lang (Lattice QCD for Pedestrians); 2008

Motivation

Problems that cannot be attacked with perturbation theory:

- Chiral symmetry breaking
  - Explicit: Non-zero quark masses
  - Spontaneous: The pion is a Goldstone boson
- Confinement and the low energy properties of hadrons
  - Hadron masses
  - Low energy parameters (decay constants, current quark masses, LEC of Chiral Perturbation Theory)
  - Form factors, matrix elements, structure functions



We need non-perturbative methods!

- Deconfinement and chiral restoration at finite (but not large) T

QCD on the lattice (μ=0)

Partition function $$Z = \sum_n \langle n | e^{-\hat{H}/T} | n \rangle$$

Can be expressed, in the Feynman path integral approach:

$$Z = \int [dA\, d\bar{\psi}\, d\psi] e^{-\int_0^{1/T} d\tau \int d^3x \mathcal{L}_{QCD}}$$

where $$\mathcal{L}_{QCD} = \frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} + \sum_f \bar{\psi}_f (\not{D} + m_f) \psi_f$$ Eucl. Space-time

With $D_\mu = \partial_\mu + i A_\mu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]$

Finite T: Imaginary time 0 -> 1/T with periodic boundary conditions

Gauge invariance of the action under

$$\psi(x) \rightarrow \psi'(x) = \Omega(x)\psi(x)$$

$$\overline{\psi}(x) \rightarrow \overline{\psi}'(x) = \overline{\psi}(x)\Omega(x)^\dagger$$

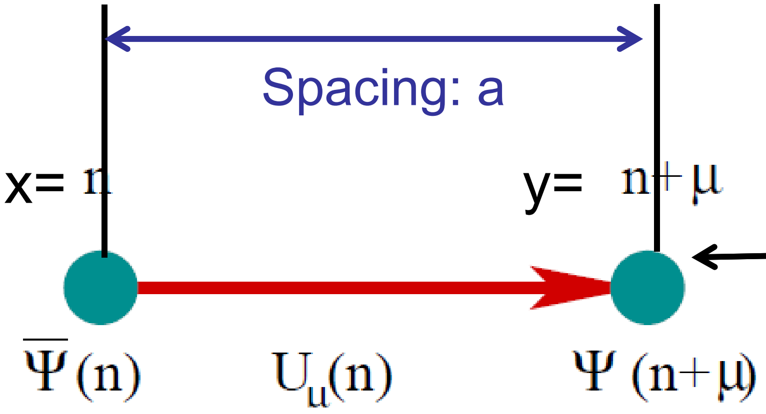
$$A_\mu(x) \rightarrow A'_\mu(x) = \Omega(x) A_\mu(x) \Omega(x)^\dagger + \mathrm{i}(\partial_\mu \Omega(x))\Omega(x)^\dagger$$

QCD on the lattice (μ=0)

Going to the lattice (Wilson)

$$\mathcal{L}_{QCD} = \frac{1}{2g^2}\mathrm{Tr}F_{\mu\nu}F_{\mu\nu} + \sum_f \bar{\psi}_f(\not{D} + m_f)\psi_f$$

$$D_\mu = \partial_\mu + iA_\mu$$



At each node: ψ: Grassman variable (4 x Nc x Nf)

In continuous theory : Link $G(x,y) = P\exp\left(i\int_{\mathcal{C}_{xy}} A \cdot ds\right)$

On the lattice: $U_\mu(n) \approx 1 + iaA_\mu(n)$ with $U_\mu(n) \rightarrow U'_\mu(n) = \Omega(n)\,U_\mu(n)\,\Omega(n+\hat{\mu})^\dagger$

$$S_F \rightarrow a^4 \sum_n \bar{\psi}(n)\left[\sum_\mu \gamma_\mu \frac{U_\mu(n)\psi(n+\mu) - \psi(n)}{a} + m\psi(n)\right]$$

and $\{A\} \rightarrow \{U\}$

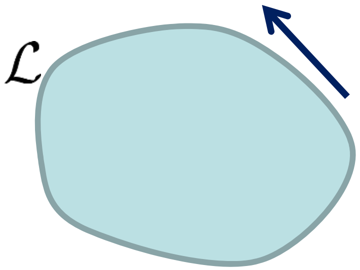
Change of dof for the gluonic fields

QCD on the lattice (μ=0)

Going to the lattice (Wilson)

$$\mathcal{L}_{QCD} = \frac{1}{2g^2}\mathrm{Tr}F_{\mu\nu}F_{\mu\nu} + \sum_f \bar{\psi}_f(\not{D} + m_f)\psi_f$$

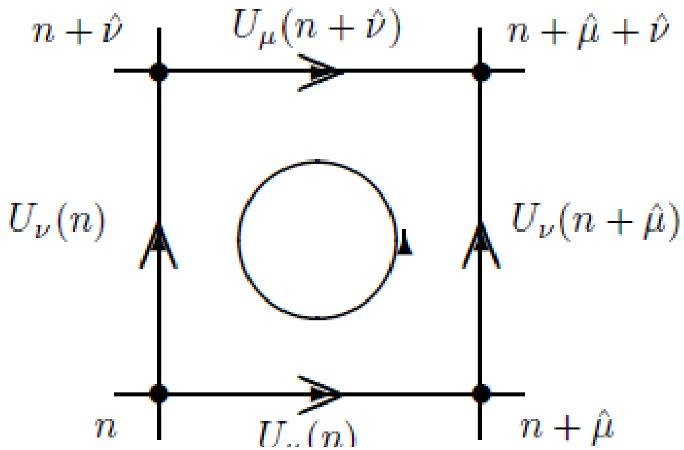
In continuous theory : $P\exp\left(i\oint_{\mathcal{L}} A_\mu dl^\mu\right)$ is gauge invariant



On the lattice $L[U] = \mathrm{Tr}\left[\prod_{(n,\mu)\in\mathcal{L}} U_\mu(n)\right]$ as $U_\mu(n) \to U'_\mu(n) = \Omega(n)\,U_\mu(n)\,\Omega(n+\hat{\mu})^\dagger$

Simplest choice for $\mathcal{L}$: plaquette

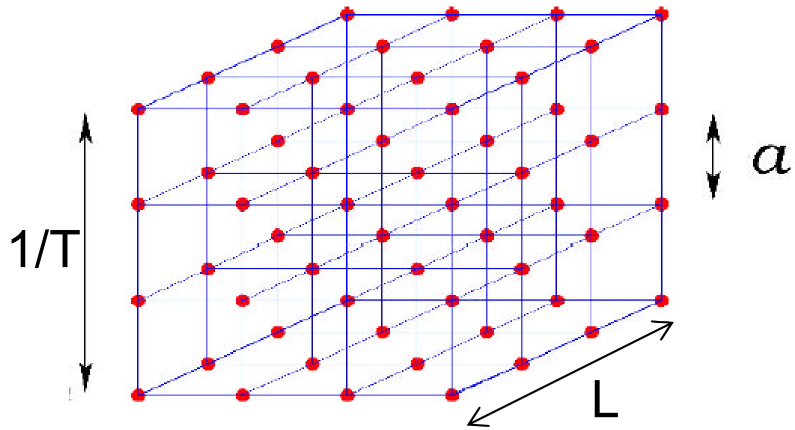
$$U_{\mu\nu}(n) = U_\mu(n)\,U_\nu(n+\hat{\mu}) \times U_{-\mu}(n+\hat{\mu}+\hat{\nu})\,U_{-\nu}(n+\hat{\nu})$$



$$S_G[U] = \frac{2}{g^2}\sum_{n\in\Lambda}\sum_{\mu<\nu}\mathrm{Re\,Tr}\left[1-U_{\mu\nu}(n)\right] = \frac{a^4}{2g^2}\sum_{n\in\Lambda}\sum_{\mu,\nu}\mathrm{Tr}\left[F_{\mu\nu}(n)^2\right] + \mathcal{O}(a^2)$$

QCD on the lattice (μ=0)

In practice:



- Lattice is also a method for regularisation and renormalisation
- Fermionic fields are Grassman variables on the nodes: cannot be simulated (efficiently) with numerical methods => integrate by hand and generate large dets' on *U*
- Gauge invariant formulation, reproduces the continuum limit when a ->0
- Evaluation of extra observables

$$\langle O_2(t)\, O_1(0)\rangle = \frac{1}{Z}\int \mathcal{D}[\psi,\overline{\psi}]\,\mathcal{D}[U]\, e^{-S_F[\psi,\overline{\psi},U]-S_G[U]}\, \underbrace{O_2[\psi,\overline{\psi},U]\, O_1[\psi,\overline{\psi},U]}$$

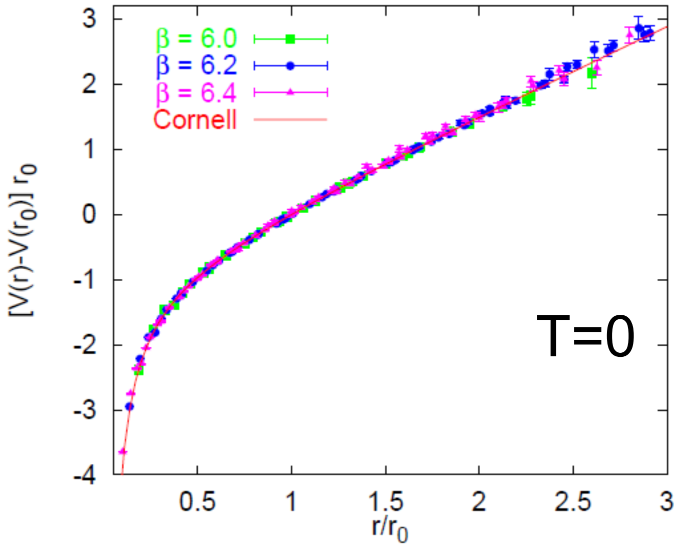
Need to be Gauge invariant
(Wilson loop, Polyakov loop)

- Evaluated thanks to Monte Carlo methods

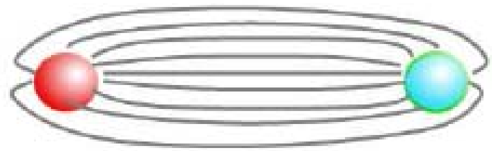
QCD on the lattice (μ=0)

In practice:

- Fixing parameters ($a(\beta=6/g^2)$, masses): compare with force at a fixed value r_0 (quarkonium spectroscopy) and with the hadrons masses (a down to 0.05fm)



Confirms the string picture

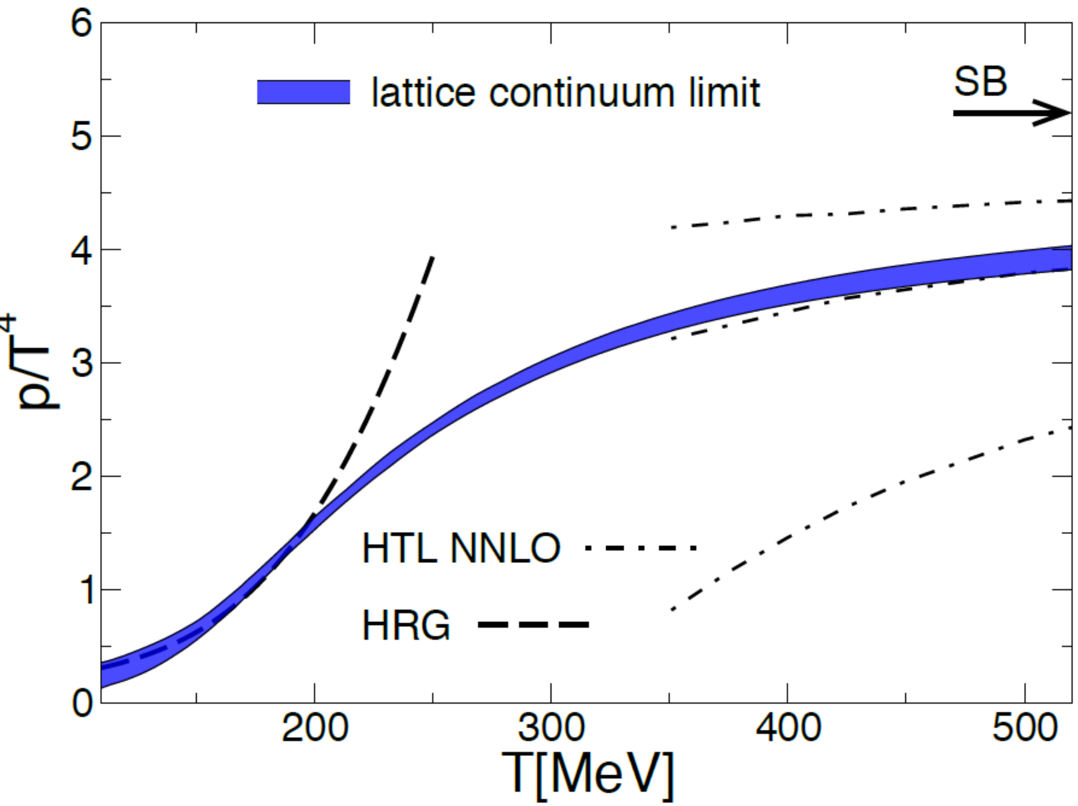


- Need to go to the continuous limit (a->0) the thermodynamic limit ($L \rightarrow \infty$) and the "chiral" limit ($m_\pi = m_{\pi exp}$)

Standard results from lQCD

From **Borsányi et al.**, arXiv:1312.2193v1 (hep-lat) ← Not hep-nuc, not even hep-th

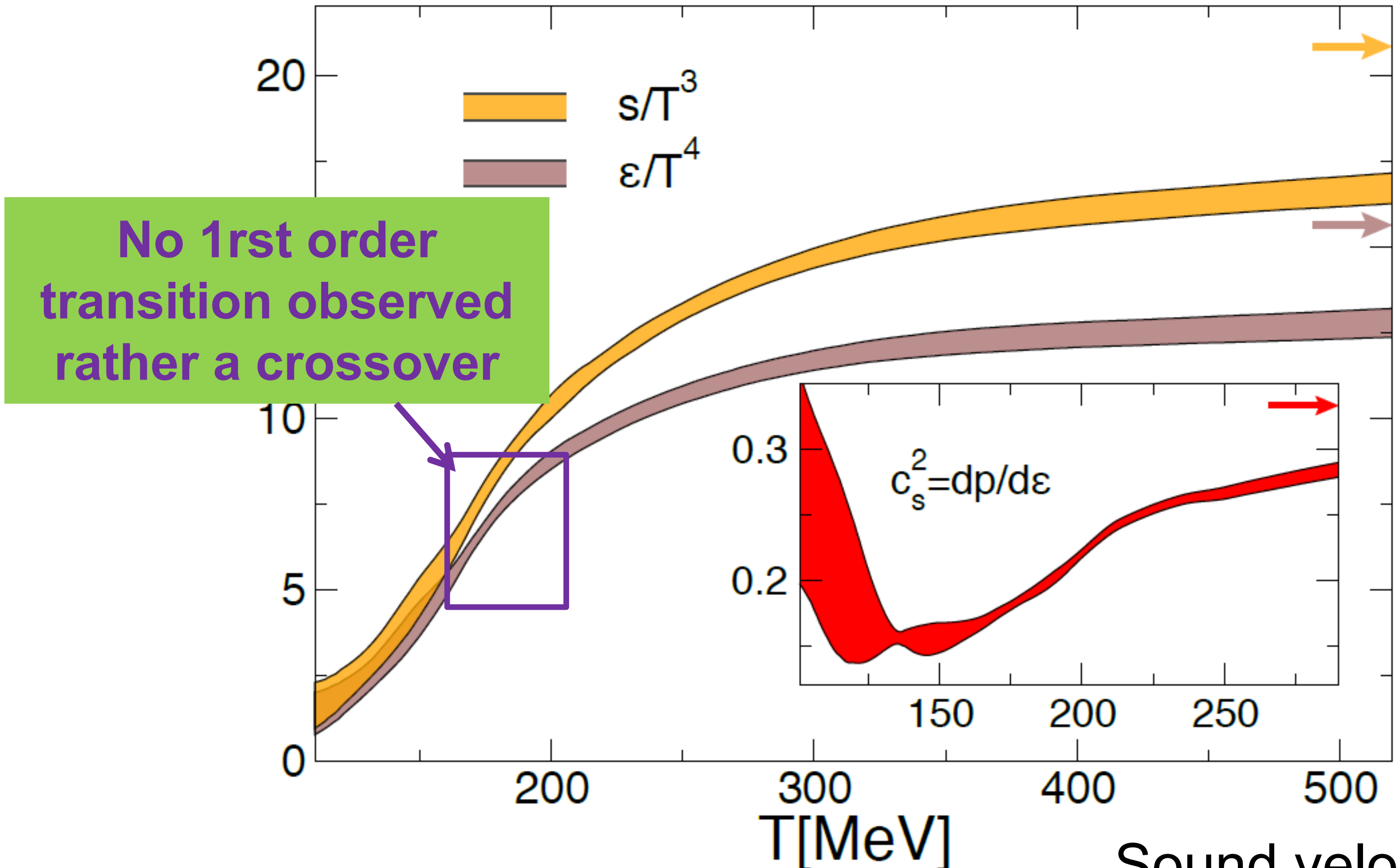
One recovers the Hadron Resonance Gas EOS at small T

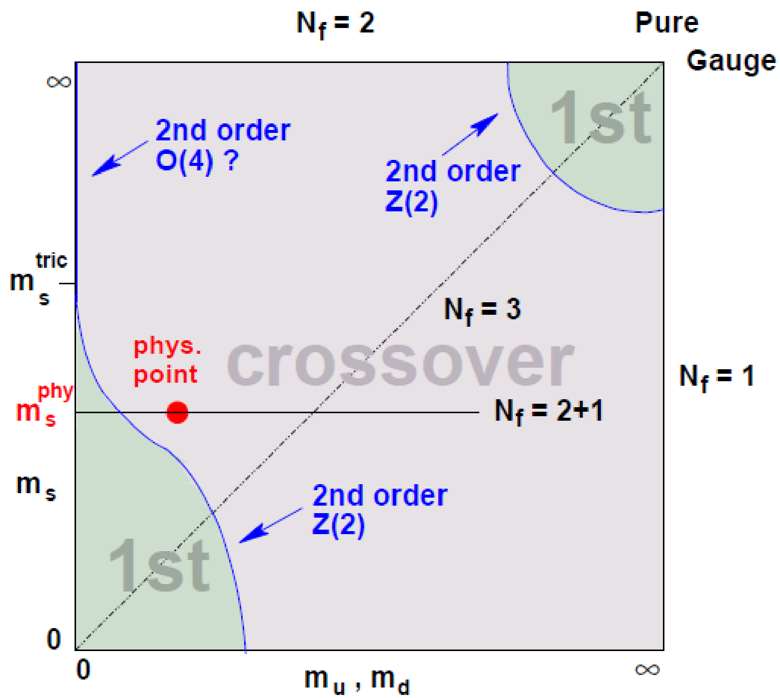


significant deviations wrt SB at the highest T: collect. excitations ?

Standard results from lQCD at μ=0

From **Borsányi et al.,** arXiv:1312.2193v1 (hep-lat)





possible prescription: Tc ≡ inflexion point

≈ 160 MeV

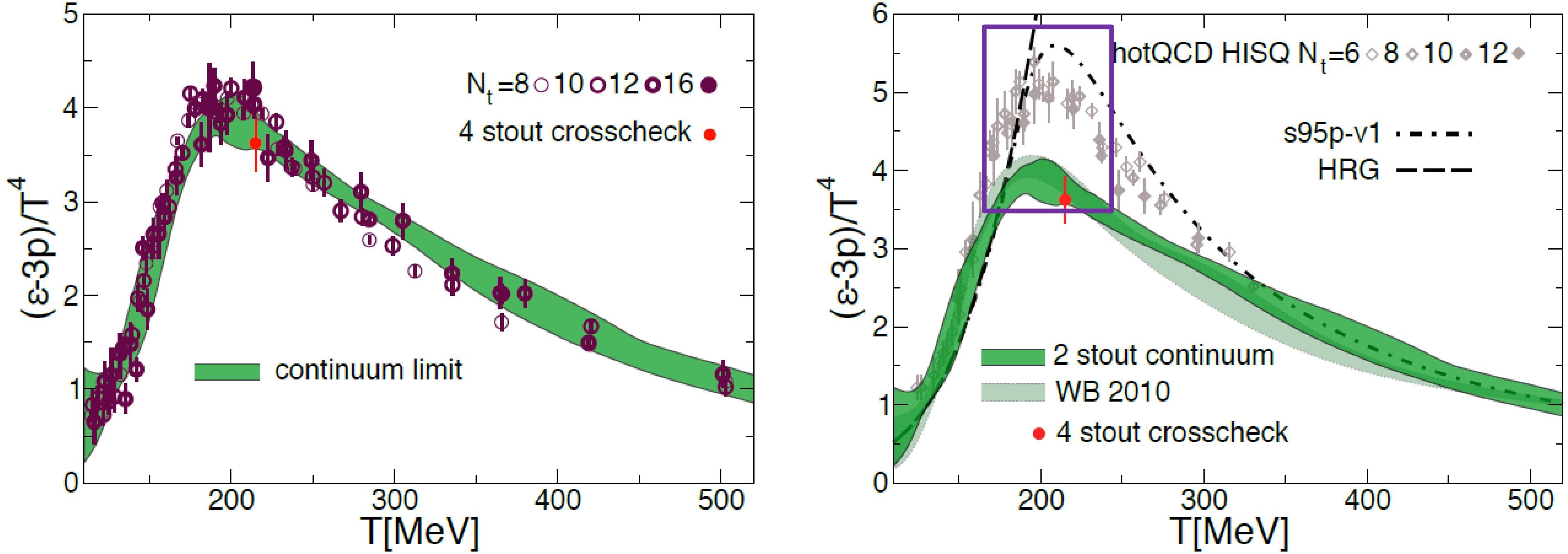
Sound velocity does not vanish at the "transition"

Standard results from lQCD

Consistency checks:

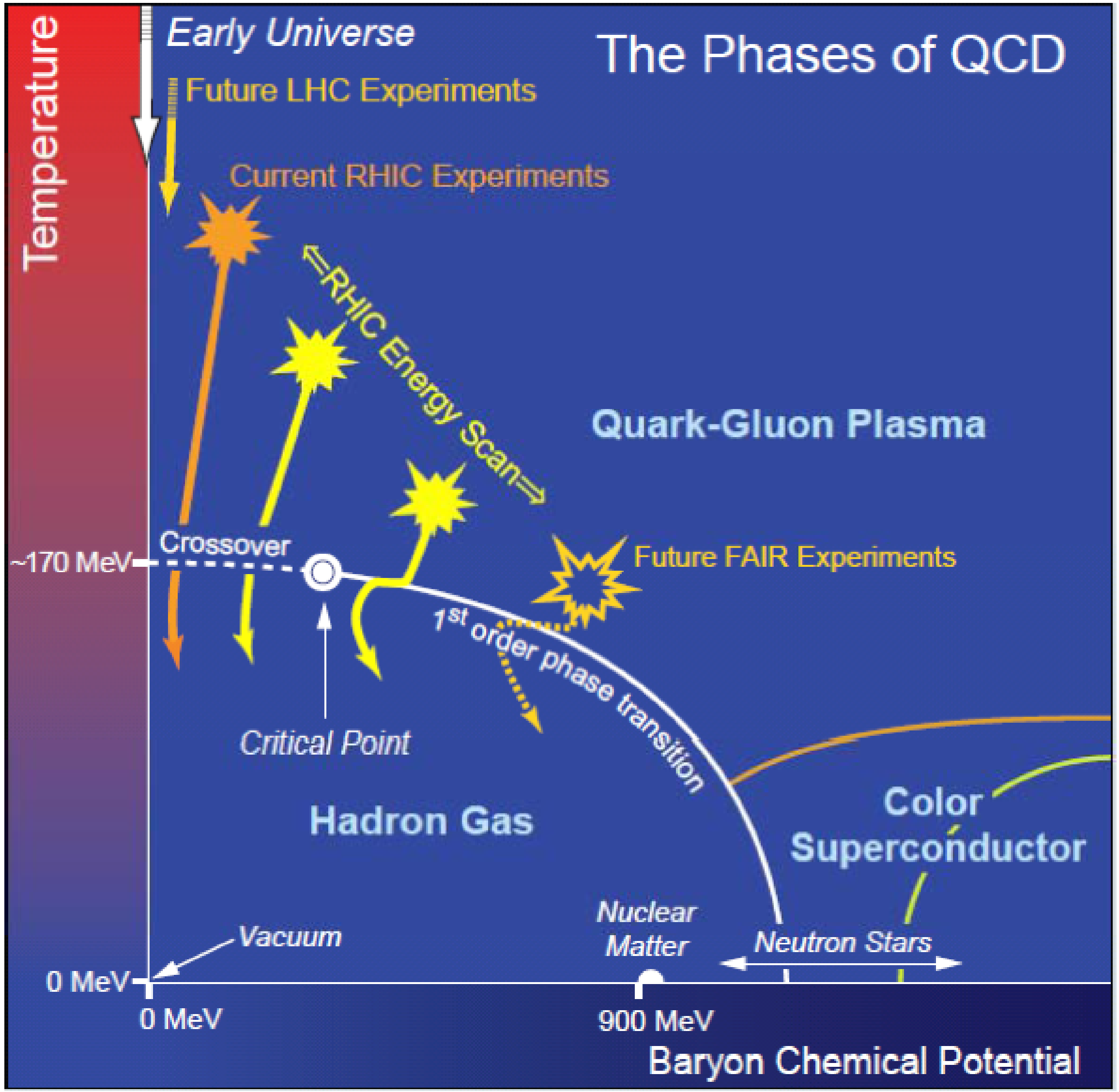
From **Borsányi et al.**, QM 2012 presentation & arXiv:1312.2193v1 (hep-lat)

chiral Tc	Budapest-Wuppertal	BNL-Bielefeld	MILC
2004			169(14)(4)
2006	151(3)(3)	192(4)(7)	
2009	146(2)(3) - 157(3)(3)		
2010	147(2)(3) - 155(3)(3)		
2012		154(8)(1)	



≈ 20% residual uncertainty between various groups

lQCD at finite μ



lQCD at finite μ

Bulk thermodynamics with non-vanishing chemical potential

From F. Karsch (Lattice results on the QCD critical point); Seattle 2008

$$Z(V,T,\mu) = \int \mathcal{D}\mathcal{A}\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \mathrm{e}^{-S_E(V,T,\mu)}$$
$$= \int \mathcal{D}\mathcal{A}\ \left[det\ M(\mu)\right]^f\ \mathrm{e}^{-S_G(V,T)}$$

⇑complex fermion determinant;

Monte Carlo translates weight $\exp(-S_E)$ into probability and fails if S_E is not real.

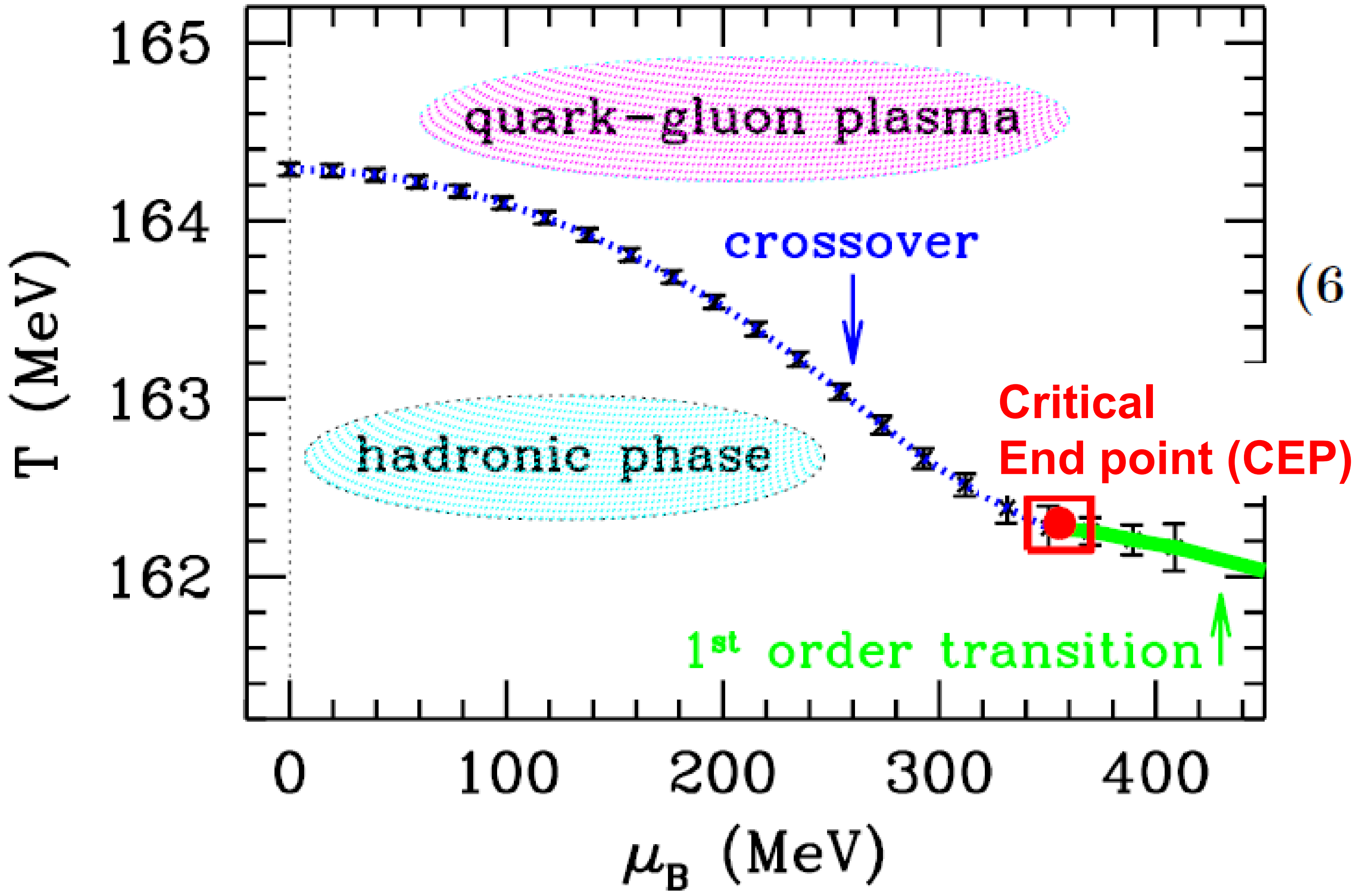
ways to circumvent this problem:

- reweighting: works well on small lattices; requires exact evaluation of $det M$
  Z. Fodor, S.D. Katz, JHEP 0203 (2002) 014
- Taylor expansion around $\mu = 0$: works well for small μ;
  C. R. Allton et al. (Bielefeld-Swansea), Phys. Rev. D66 (2002) 074507
  R.V. Gavai, S. Gupta, Phys. Rev. D68 (2003) 034506
- imaginary chemical potential: works well for small μ; requires analytic continuation
  Ph. deForcrand, O. Philipsen, Nucl. Phys. B642 (2002) 290
  M. D'Elia and M.P. Lombardo, Phys. Rev. D64 (2003) 014505
- canonical ensemble: need to evaluate fermion determinant
  K.-F. Liu, Int. J. Mod. Phys. B16 (2002) 2017
  S. Kratochvila and P. de Forcrand, PoS LAT2005, 167 (2006)

lQCD at finite μ

From F. Karsch (Lattice results on the QCD critical point); Seattle 2008

reweighting:



$(6-12)^3 \times 4$

Z. Fodor, S. Katz, JHEP 0404 (2004) 050

Bulk thermodynamics for small μ_q/T

$$\frac{p}{T^4} = \frac{1}{VT^3}\ln \mathcal{Z} = \sum_{n=0}^{\infty} c_n(T)\left(\frac{\mu_q}{T}\right)^n \simeq c_0 + c_2\left(\frac{\mu_q}{T}\right)^2 + c_4\left(\frac{\mu_q}{T}\right)^4 + c_6\left(\frac{\mu_q}{T}\right)^6$$

quark number density
$$\frac{n_q}{T^3} = 2c_2\frac{\mu_q}{T} + 4c_4\left(\frac{\mu_q}{T}\right)^3 + 6c_6\left(\frac{\mu_q}{T}\right)^5$$

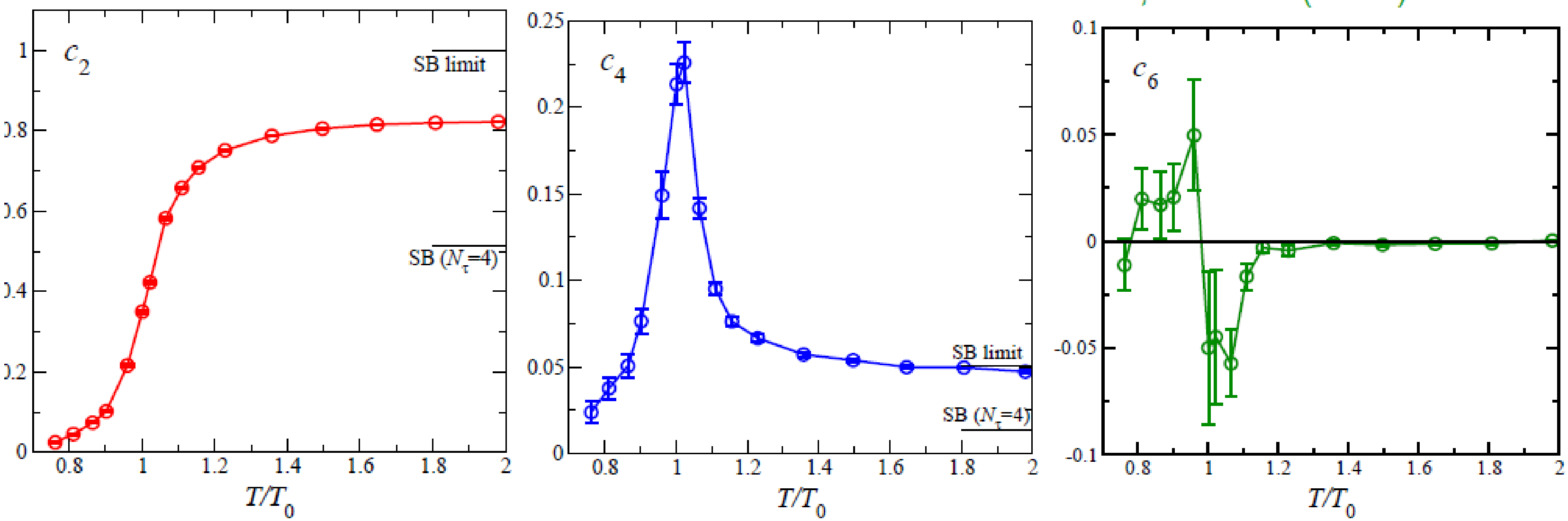
Not the only one, inspiration from HG models

an estimator for the radius of convergence

$$\left(\frac{\mu_q}{T}\right)_{crit} = \lim_{n\to\infty}\left|\frac{c_{2n}}{c_{2n+2}}\right|^{1/2}$$

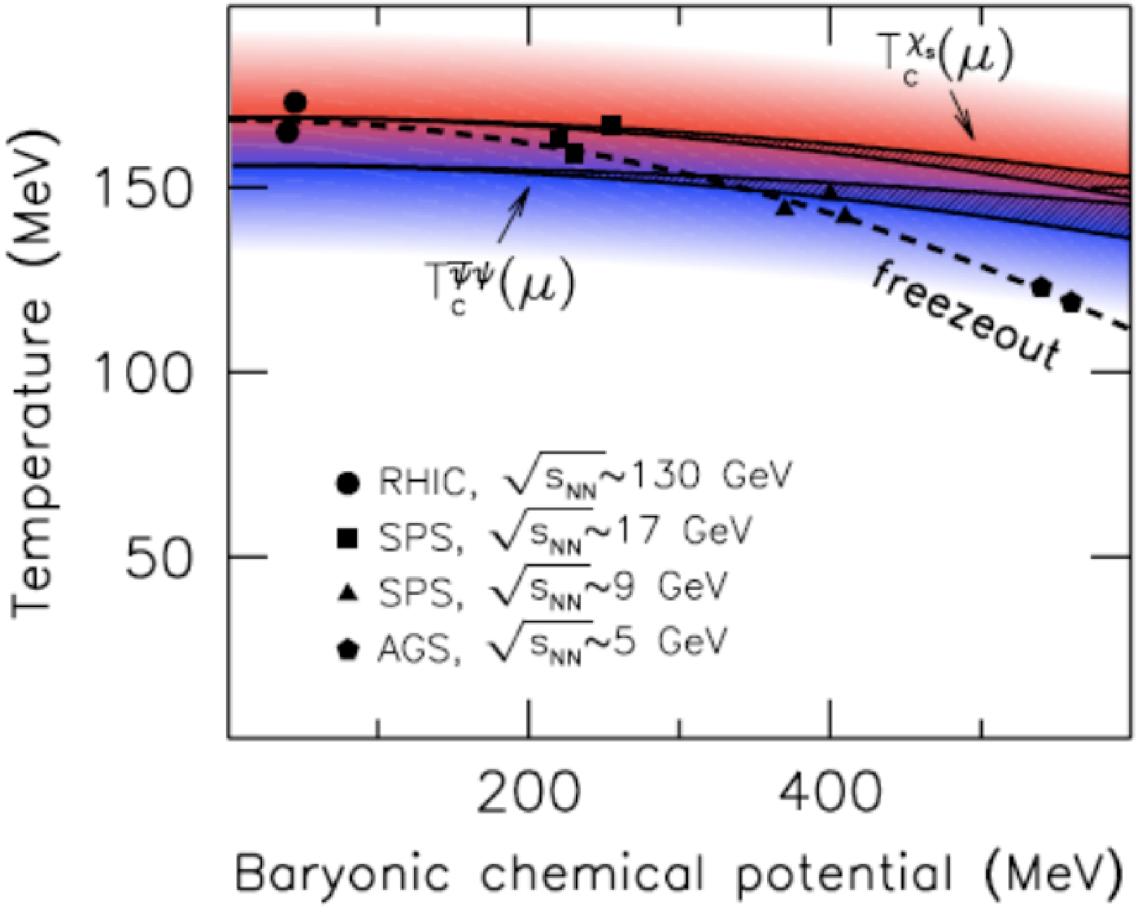
$c_n > 0$ for all n
$\Rightarrow$ singularity for real μ

C. Allton et al, PR D71 (2005) 054508



Estimating the curvature of the crossover line

$$\kappa = -T_c \left. \frac{dT_c(\mu^2)}{d(\mu^2)} \right|_{\mu=0} \approx 0.0066 - 0.0089$$



Freeze out line (AA exp.) has larger curvature then crossover line

Confirms that white hadrons stay in equilibrium during expansion ?

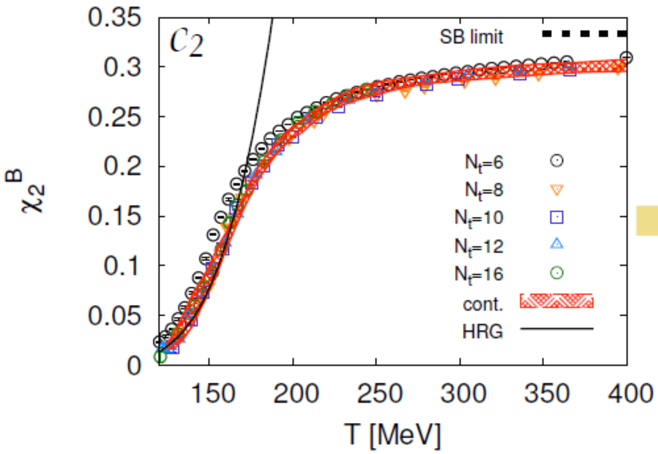
Szablocs Borsanyi (QM 2012)

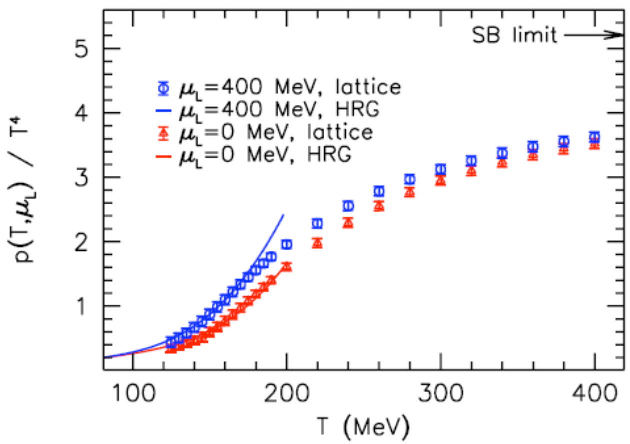
EOS at finite μ

Szablocs Borsanyi (QM 2012)

Taylor approach:

$$\frac{p}{T^4} \equiv \frac{1}{VT^3} \ln Z(T,\mu_q) = \sum_{n=0}^{\infty} c_n(T,m_q) \left(\frac{\mu_q}{T}\right)^n \quad with \quad c_n(T,m_q) = \frac{1}{n!}\frac{1}{VT^3}\frac{\partial^n \ln Z(T,\mu_q)}{\partial(\mu_q/T)^n}\bigg|_{\mu_q=0}$$





Budapest-Wuppertal:

use continuum free energy + continuum 1st derivative in $\left(\frac{\mu}{T}\right)^2$

[Wuppertal-Budapest 1204.6710]

MILC/BNL-Bielefeld:

coarse lattice (Nt=6) + 3rd order expansion in $\left(\frac{\mu}{T}\right)^2$

[De Tar et al 1003.5682] [Ejiri et al hep-lat/0512040]

use s95p parametrization (asqtad Nt=8) + c2 (HISQ Nt=8)
+ c4 (p4, Nt=4, shifted) + c6 (p4, Nt=4, shifted)

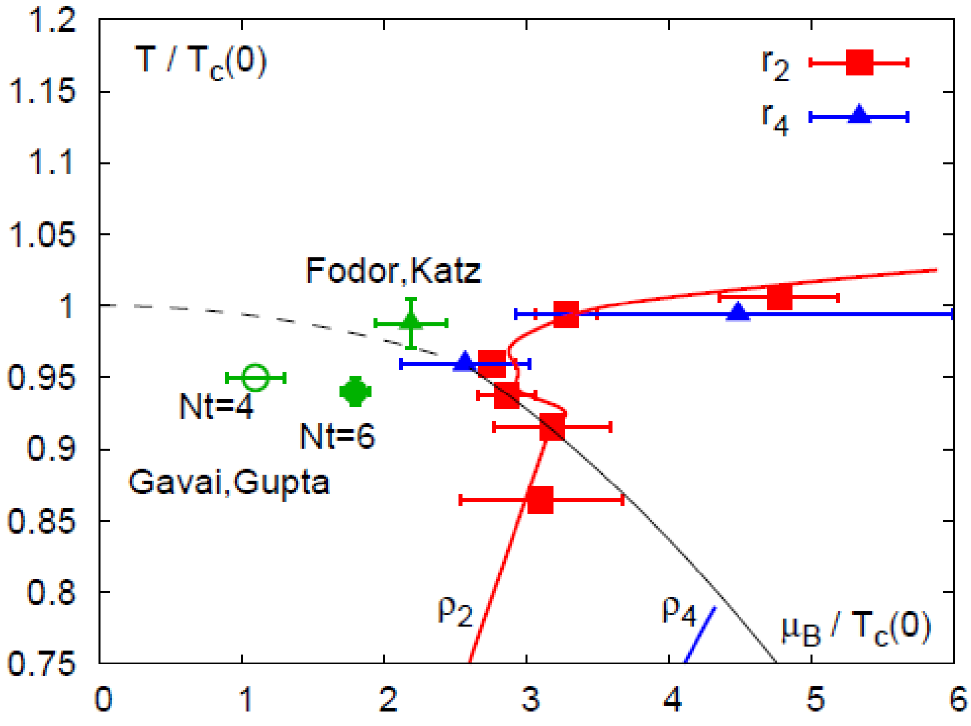
[Huovinen&Petreczky &Schmidt, 2011]

Estimating the critical point (the end Taylor)

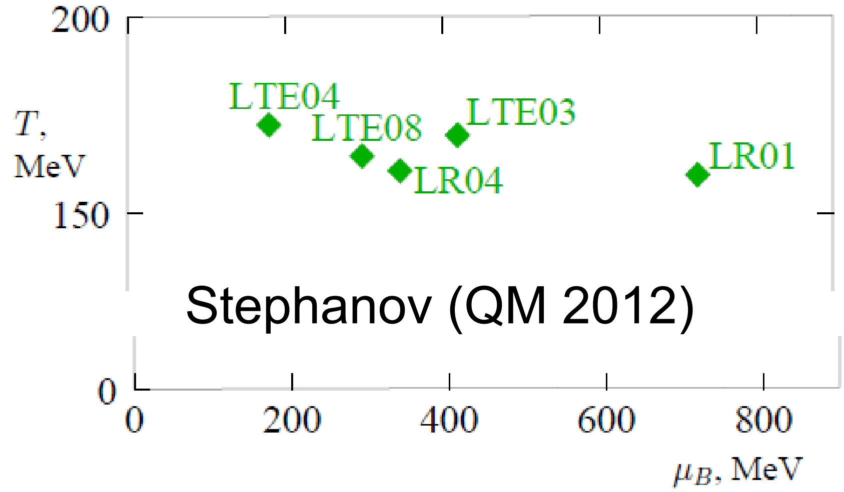
Status of the RBC-BI project

From F. Karsch (Lattice results on the QCD critical point); Seattle 2008

- calculations for $N_\tau = 4$ and 6; $N_\sigma = 4N_\tau$
- uses an $\mathcal{O}(a^2)$ improved staggered action (p4fat3)
- estimator for μ_c: $$\left(\frac{\mu_c(T)}{T_c(0)}\right)_n \equiv \rho_n = \frac{T}{T_c(0)}\sqrt{\frac{c_n}{c_{n+2}}}$$



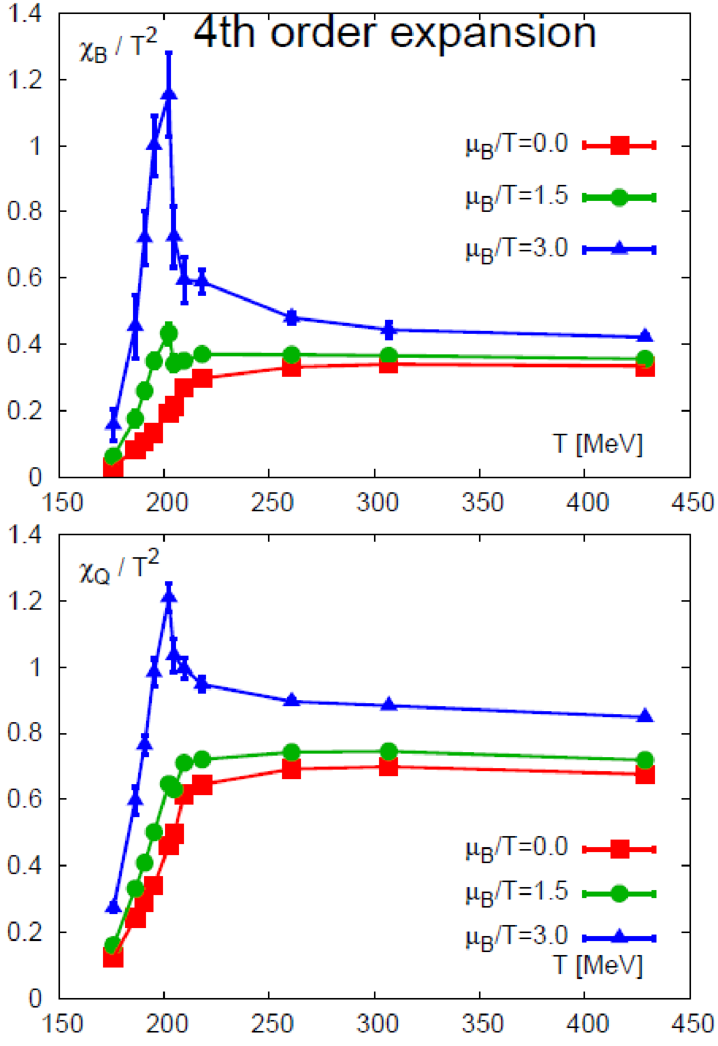
- slight quark mass dependence
- weak cut-off dependence



Estimating the critical point: contact with the experiments

From F. Karsch (Lattice results on the QCD critical point); Seattle 2008

Fluctuations of baryon number and charge densities ($\mu \geq 0$)



baryon number density fluctuations:

$$\frac{\chi_B}{T^3} = \left(\frac{\mathrm{d}^2}{\mathrm{d}(\mu_B/T)^2}\frac{p}{T^4}\right)_{T\ \mathrm{fixed}}$$

$$= \frac{T}{V}\left(\langle N_B^2\rangle - \langle N_B\rangle^2\right)$$

susceptibilities

- to be studied in event-by-event fluctuations
- to be compared to hadron resonance gas phenomenology

seeing "true" singular behavior as signal for a critical point requires large volumes and high order Taylor expansions

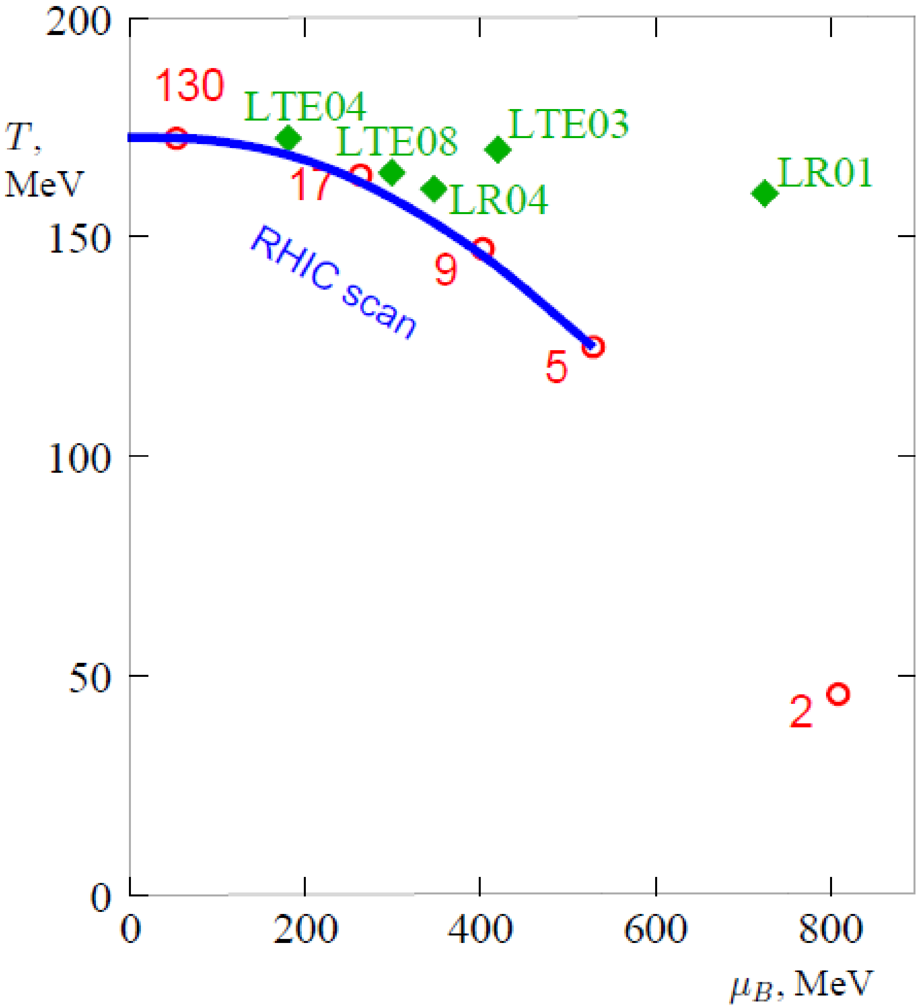
$m_\pi = 220$ MeV, (2+1)-flavor QCD

evidence for a critical point??

INT, Seattle 2008, F. Karsch – p. 30/32

Estimating the critical point: contact with the experiments

Location of the critical point vs freeze-out



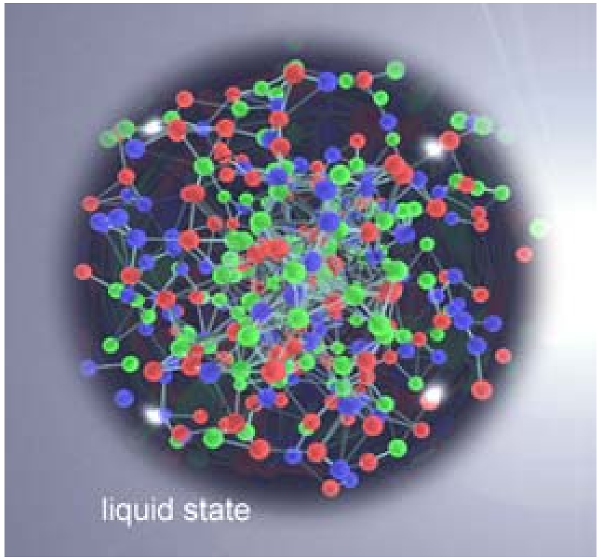
To do:

- Experiments:
  - RHIC,
  - NA61(SHINE) @ SPS,
  - CBM @ FAIR/GSI
  - NICA @ JINR
- Improve lattice predictions, understand systematic errors.
- Find most sensitive/optimal signatures and understand the effects of the dynamics of a h.i.c. on them.

Stephanov (QM 2012)

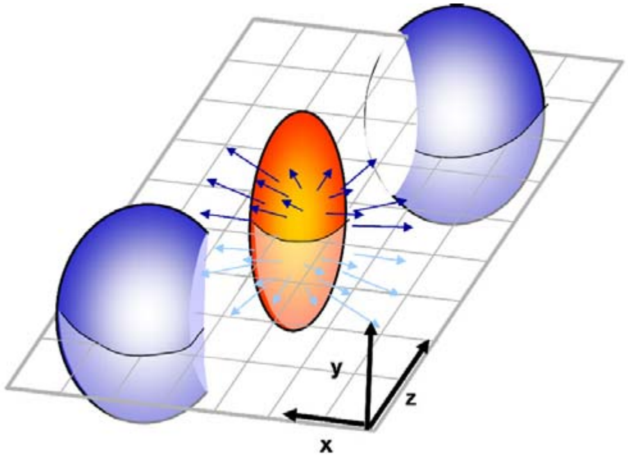
Less standard questions to lQCD

- Is it “weakly coupled” or “strongly coupled” ?
- Is it a gas or a fluid ?
- What about other properties (transport coefficients)
- Are there some effective degrees of freedom ?
- ….



Weakly or Strongly ?

Slides 34 → 38: Some elements from W.A Zajc (The Quest for the QGP) QM 2012



- Is it “weakly coupled” or “strongly coupled” ?

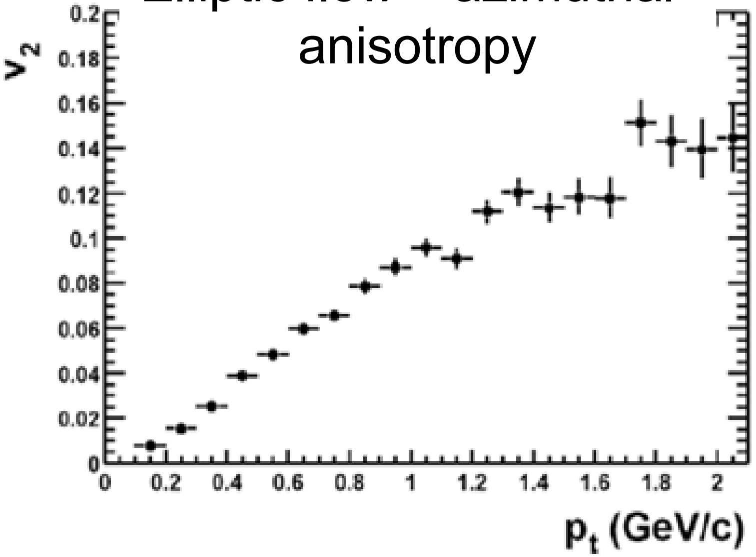
One should not be confused by the word “plasma” !!!

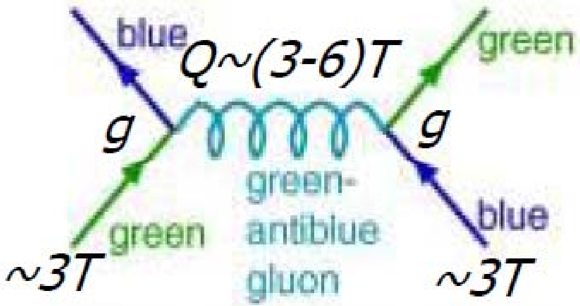
At RHIC (& LHC): discovery of large flows and large “jet quenching” => s(trong)QGP (?)

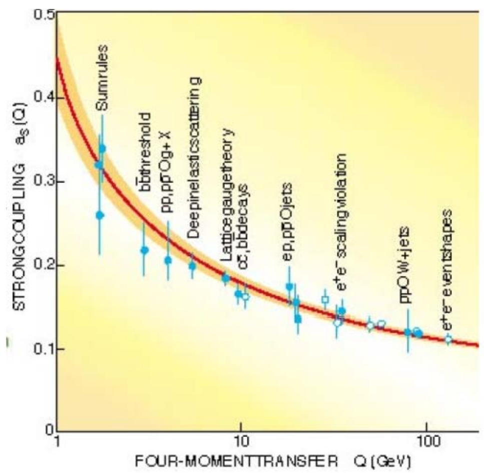
“Real” plasma physicists always want to know the value of the classical coupling parameter Γ:

$$\Gamma \equiv \frac{<Potential\ Energy>}{<Kinetic\ Energy>} = \frac{Debye\ Mass}{<Kinetic\ Energy>} \sim \frac{gT}{3T} \sim 1$$

Elliptic flow ≡ azimuthal anisotropy

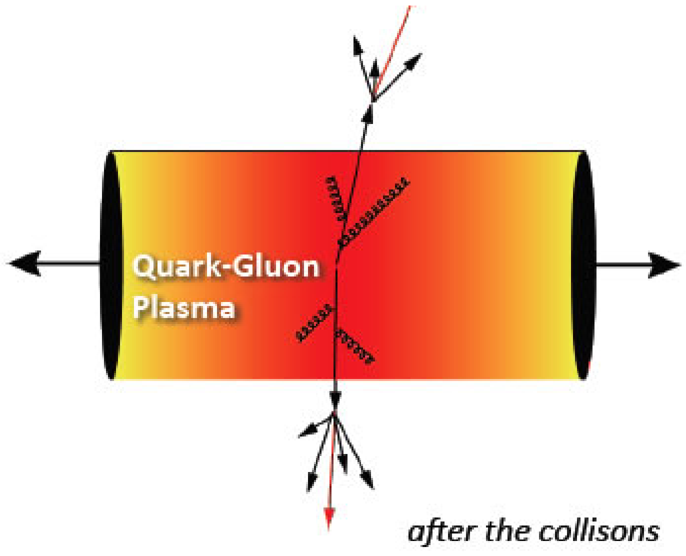






Weakly or Strongly ?

➢ Is it "weakly coupled" or "strongly coupled" ?



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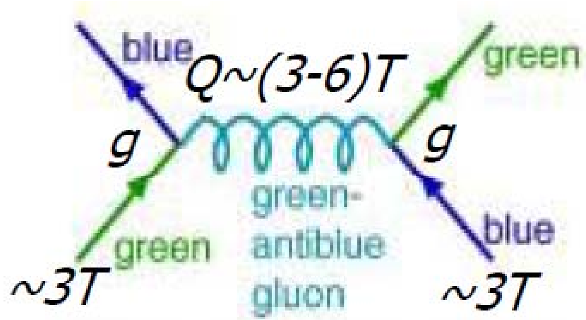
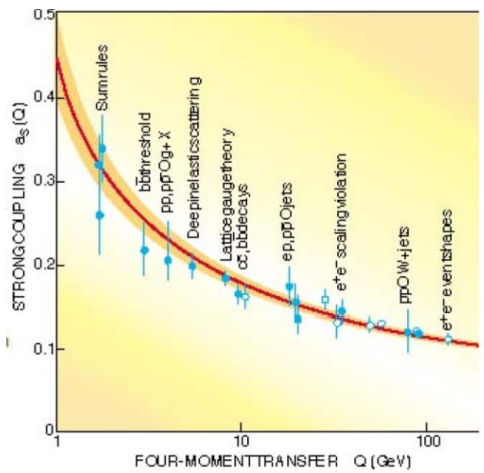
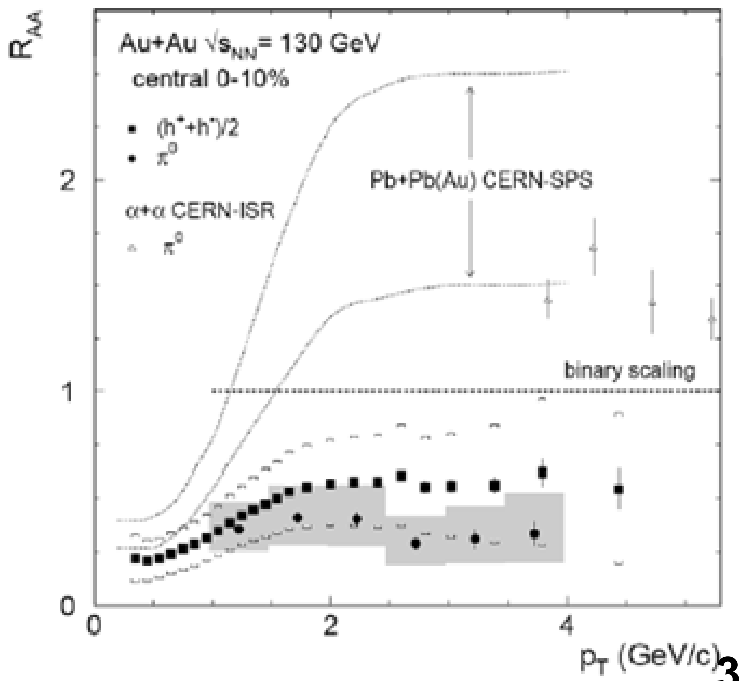






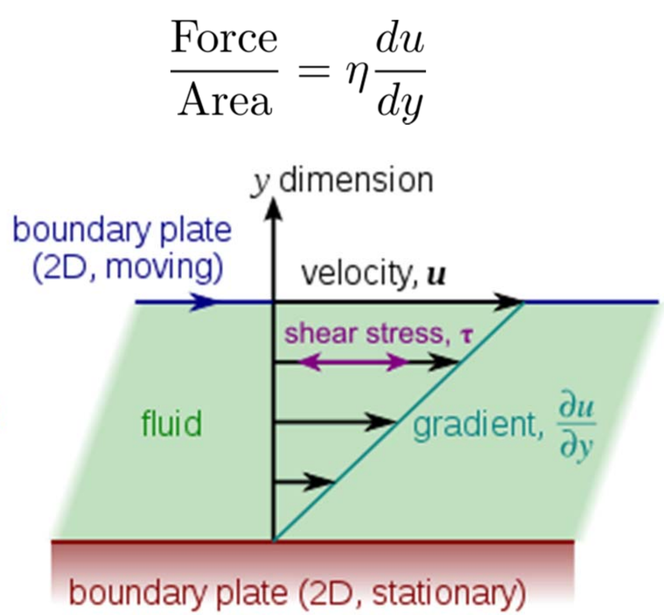
Weakly or Strongly ?

- Is it "weakly coupled" or "strongly coupled" ?

On the particle level : strongly coupled ≡ large cross sections σ ≡ local equilibration

On fluid dynamics level: large cross sections σ ≡ "small" transport coefficients

- Viscosity η ~ Transverse momentum diffusion
  ~ n <p> λ , n = number density
  , <p> = mean momentum
  , λ = mean free path
  = 1 / nσ ,
  σ = cross section



- So

$$\eta \sim n \langle p \rangle \lambda \sim \langle p \rangle / \sigma$$

- *Large* interparticle cross section ➔ *small* viscosity

Weakly or Strongly ?

- Is it "weakly coupled" or "strongly coupled" ?
- What about other properties (transport coefficients)

To what should we compare the shear viscosity to obtain some intrinsic number ?

Navier-Stokes:
$$\frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \vec{\nabla}\right) \vec{v} = -\frac{\vec{\nabla} p}{\rho} + \underbrace{\frac{\eta}{\rho} \Delta \vec{v}}_{\text{Diffusion term}}$$

Assume some velocity profile with variations on scale 1/T $\delta v = \frac{\eta}{\rho} / \left(\frac{\hbar c}{T}\right)^2 v \delta t$

After some elementary time 1/T, relaxation is

$$\frac{\delta v}{v}[\%] = \frac{\eta}{\rho} / \left(\frac{\hbar c}{k_B T}\right)^2 \frac{\hbar}{k_B T} = \frac{\eta}{\rho c^2 / T} \times \frac{k_B}{\hbar} = \frac{\eta}{\epsilon / T} \times \frac{k_B}{\hbar} = \boxed{\frac{\eta}{s}} \times \frac{k_B}{\hbar}$$

η/s "naturally" measured in units of $\hbar / k_B$

Weakly or Strongly ?

η/s “naturally” measured in units of $\hbar/k_B$

Bounds on η/s ?

No upper bound from first principle: a system can be as weakly interacting s possible

Lower bound ?

$$\eta \sim n\langle p\rangle\lambda \quad \text{with} \quad n \sim \frac{s}{4k_B} \quad \Rightarrow \quad \frac{\eta}{s} \sim \frac{\langle p\rangle\lambda}{4k_B}$$

+ uncertainty principle: $\langle p\rangle\lambda \geq \frac{\hbar}{2}$

$$\Rightarrow \quad \frac{\eta}{s} \gtrsim \frac{\hbar}{8k_B}$$

Dissipative Phenomena In Quark-Gluon Plasmas
P. Danielewicz, M. Gyulassy
Phys.Rev. D31, 53,1985.

Rigorous lower bound:

(in some strongly coupled YM)

Viscosity in Strongly Interacting Quantum Field Theories from Black Hole Physics,
P. Kovtun, D.T. Son, A.O. Starinets,
hep-th/0405231

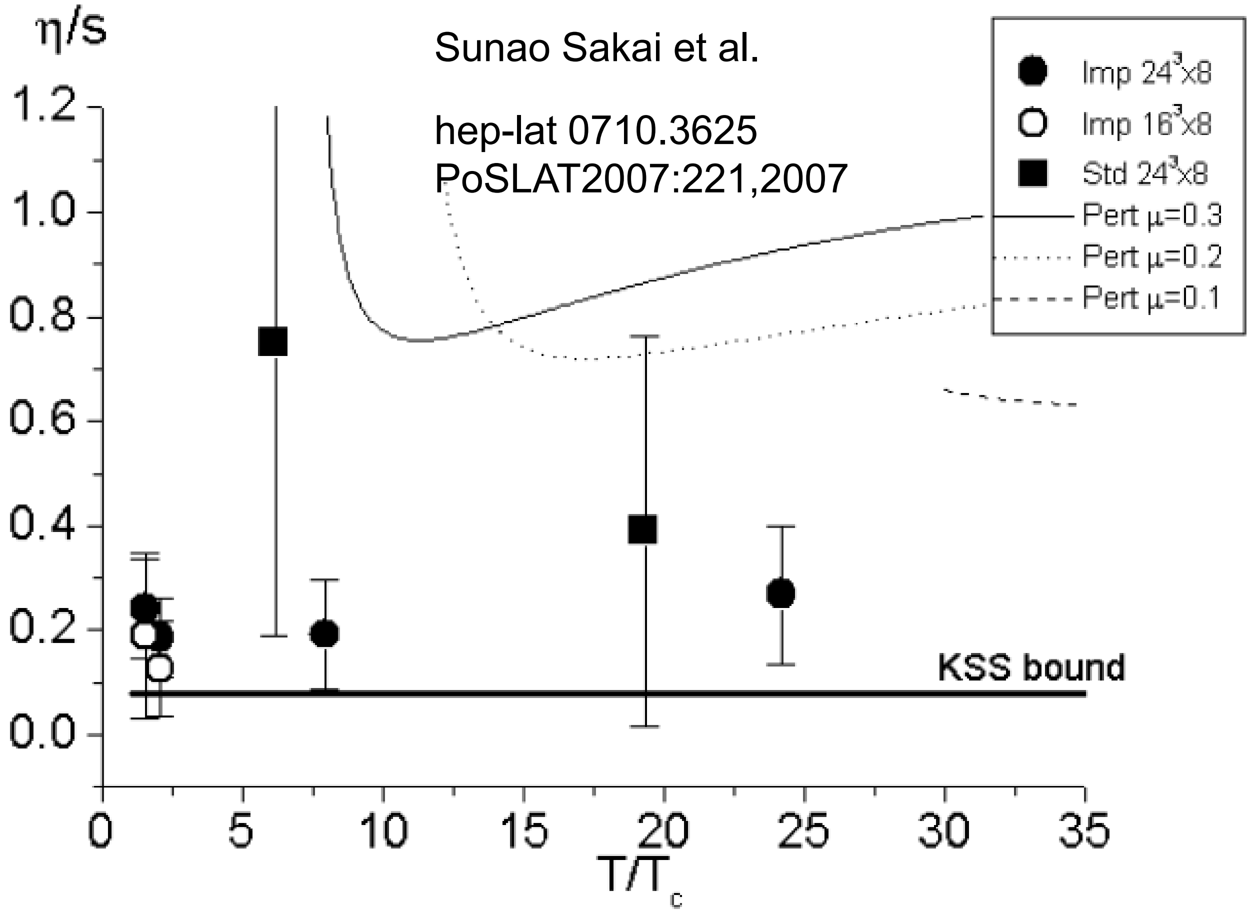
$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}$$

Weakly or Strongly ? (lQCD viewpoint)

Actual values for QCD ?

$$\eta = -\int d^3x' \int_{-\infty}^{t} dt_1 e^{\epsilon(t_1 - t)} \int_{-\infty}^{t_1} dt' \langle T_{12}(\vec{x}, t) T_{12}(\vec{x'}, t') \rangle_{ret}.$$

Kubo-like relation



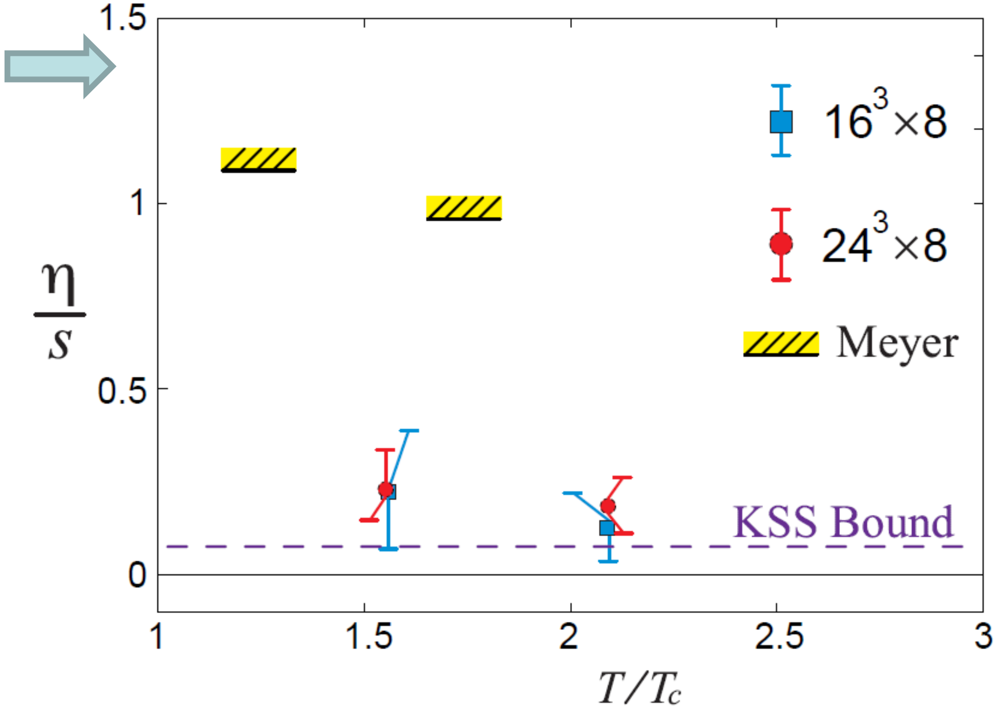
Sunao Sakai et al.

hep-lat 0710.3625
PoSLAT2007:221,2007

NLO pQCD; P. Arnold, G.D. Moore and L.G. Yaffe, *JHEP* **0305** (2003) 051.

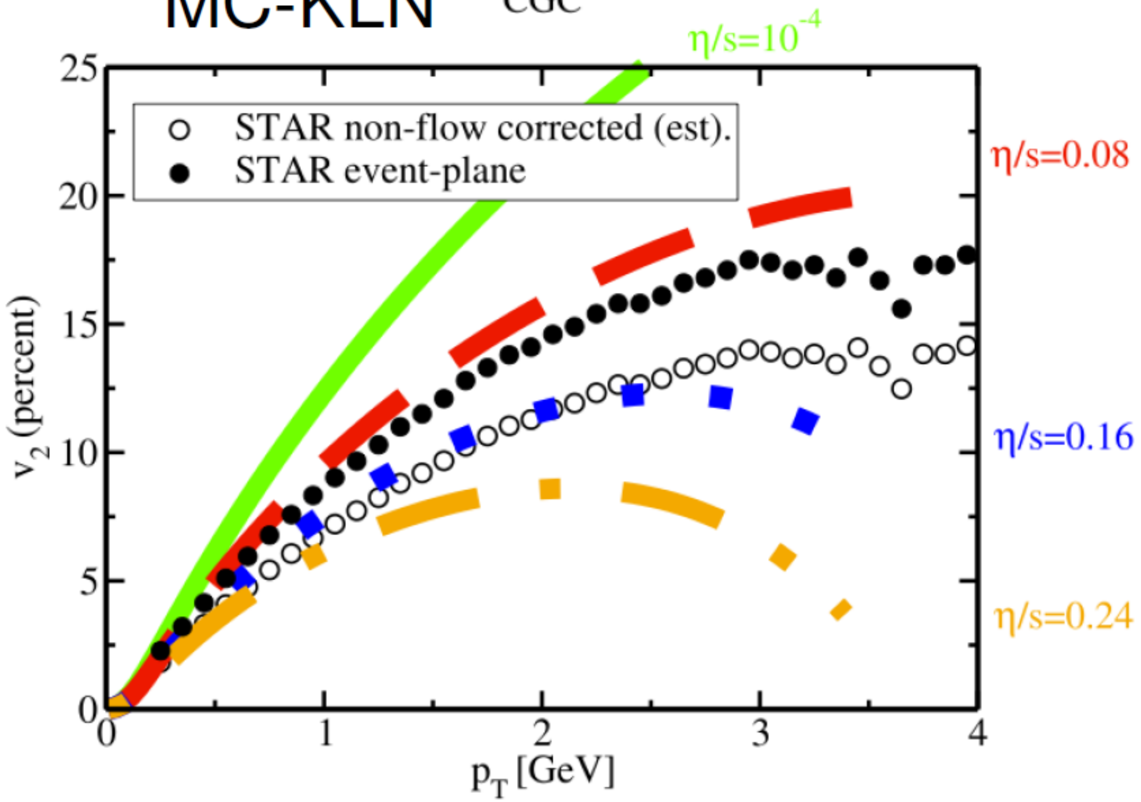
Weakly or Strongly ?

In RHIC-LHC range



Alternate approach: experiment + fluid dynamics evolution

MC-KLN CGC

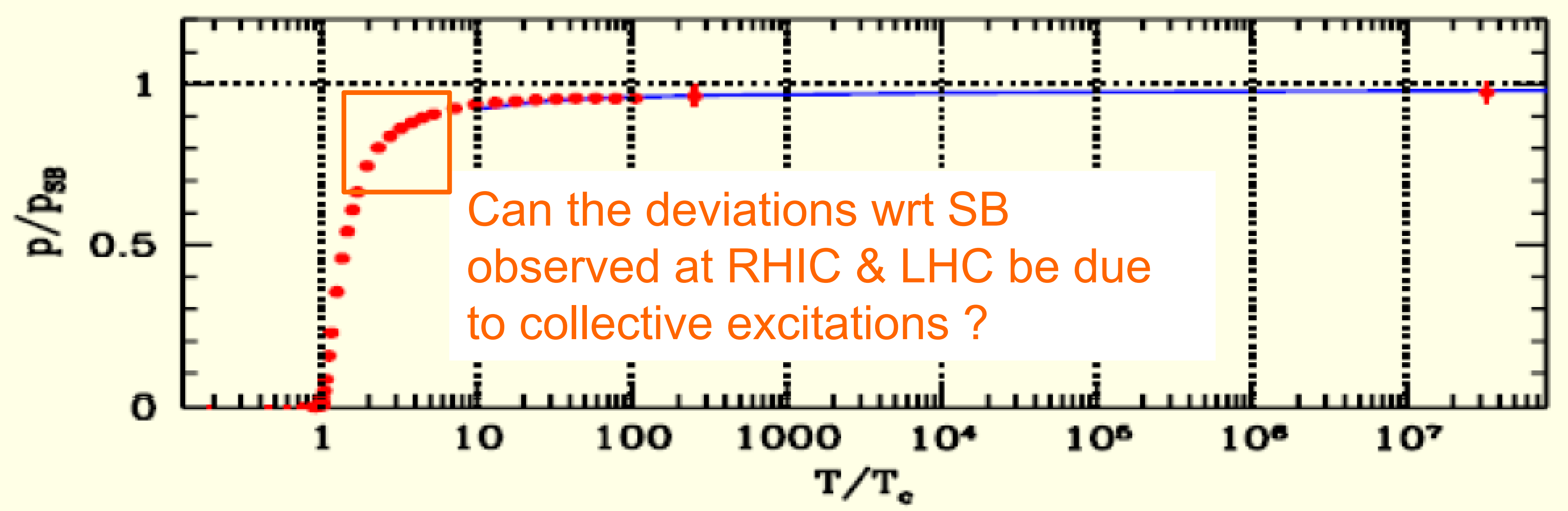


A new paradigm has emerged

- Bulk QCD matter formed at RHIC & LHC is almost infinitely coupled
- Weak coupling techniques (pQCD) is unable to cope with it

M. Luzum and P. Romatschke, Phys.Rev. C78, 034915 (2008)
Experimental data: STAR, Phys.Rev.C77, 054901 (2008)

QGP at large T



Theory framework: QFT at finite T.

Partition function $$Z = \sum_n \langle n | e^{-(\hat{H} - \mu \hat{N})/T} | n \rangle$$

Can be expressed, in the Feynman path integral approach:

$$Z = \int [dA\, d\bar{q}\, dq\, d\bar{c}\, dc] e^{-\int_0^{1/T} d\tau \int d^3x \mathcal{L}_{QCD}(\mu)}$$

Imaginary time 0 -> 1/T with periodic boundary conditions

QGP at large T

Large T limit (g=0) $$\mathcal{L}_{QCD} = \bar{q}(-i\gamma\cdot\partial + m + i\gamma_4\mu)q - \frac{1}{2}A_\mu\partial^2 A_\mu + \bar{c}\partial^2 c$$

Z_0 factorizes in well-separated contributions

Technically: Fourier decomposition of the fields

$$\phi(\tau,\vec{x}) = \sqrt{\frac{V}{T}} = \sum_{n=-\infty}^{+\infty}\sum_{\vec{k}} e^{i(\vec{k}\cdot\vec{x}-k_4\tau)}\phi_n(\vec{k})$$

Discrete Matsubara sum due to periodic BC

with $$k_4 \equiv \begin{cases} \omega_n = 2n\pi T & \text{for gluon and ghost} \\ \nu_n = (2n+1)\pi T & \text{for quark} \end{cases}$$

For instance
$$\begin{aligned} Z_0(A) &\propto \Pi_{n,\vec{k}} \int [dA_{\mu,n}(\vec{k}) e^{-\frac{1}{2}A_{\mu,n}(\vec{k})(\omega_n^2++\vec{k}^2)A_{\mu,n}(\vec{k})} \\ &\propto \Pi_{n,\vec{k}} \left[\omega_n^2 + \vec{k}^2\right]^{-2(N_c^2-1)} \end{aligned}$$

QGP at large T

differentiating wrt k

$$\frac{\partial \ln(Z_0(A)\times Z_0(c))}{\partial|\vec{k}|} = -2|\vec{k}|(N_c^2-1)\sum_{\vec{k}}\sum_{n=-\infty}^{+\infty}\frac{1}{(2\pi nT)^2+|\vec{k}|^2} = -(N_c^2-1)\sum_{\vec{k}}\frac{\coth\left(\frac{|\vec{k}|}{2T}\right)}{T}$$

$$\Rightarrow \ln(Z_0(A)\times Z_0(c)) = -2(N_c^2-1)\sum_{\vec{k}}\left[\frac{|\vec{k}|}{2T}+\ln\left(1-e^{-\frac{|\vec{k}|}{T}}\right)\right]$$

$$p_{\mathrm{glue}} = T\left.\frac{\partial \ln(Z_0(A)\times Z_0(c))}{\partial V}\right|_{T,\mu} = -2(N_c^2-1)\int\frac{d^3k}{(2\pi)^3}\left[\frac{|\vec{k}|}{2}+T\ln\left(1-e^{-E(k)/T}\right)\right]$$

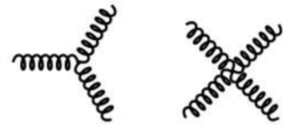
0-point energy

One indeed recovers the classical bosonic pressure with $d=2(N_c^2-1)$

$$p_B = T\left.\frac{\partial \ln Z_B}{\partial V}\right|_{T,\mu} = -d\int\frac{d^3k}{(2\pi)^3}T\ln\left(1-e^{-E(k)/T}\right)T^4$$

QGP at large T

Perturbation theory: $S = \int_0^{1/T} d\tau \int d^3x (\mathcal{L}_0 + \mathcal{L}_1) = S_0 + \underbrace{S_1}$

O(g)+O(g²):  +…

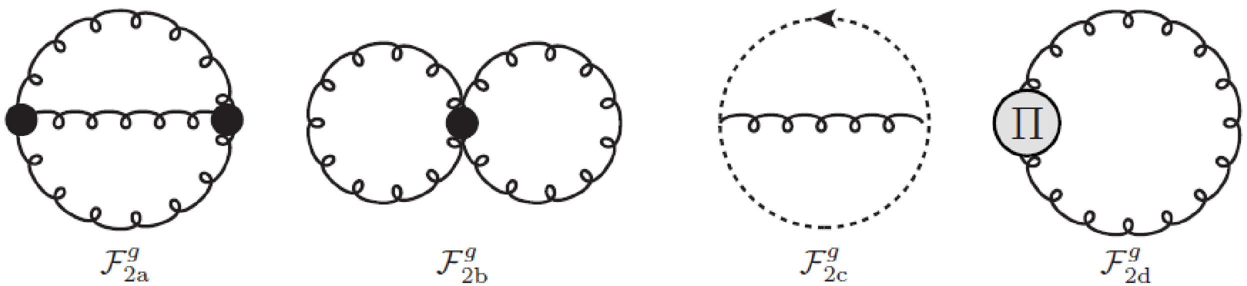
Then $\frac{Z}{Z_0} = \underbrace{\frac{\sum_{n=0}^{\infty} \frac{1}{n!} \int [d\phi](-S_1)^n \, e^{-S_0}}{\int [d\phi] e^{-S_0}}} \equiv \sum_{n=0}^{+\infty} \frac{1}{n!} \langle (-S_1)^n \rangle_0 = \exp\left[\sum_{n=1}^{\infty} \underbrace{\langle (-S_1)^n \rangle_0^c} \right]$

Similar structure to real-time Feynman path integrals

Connected Feynman diagrams with n vertices

$$\Rightarrow \ln Z = \ln Z_0 + \sum_{n=1}^{\infty} \langle (-S_1)^n \rangle_0^c$$

Example of diagrams contributing at order g²



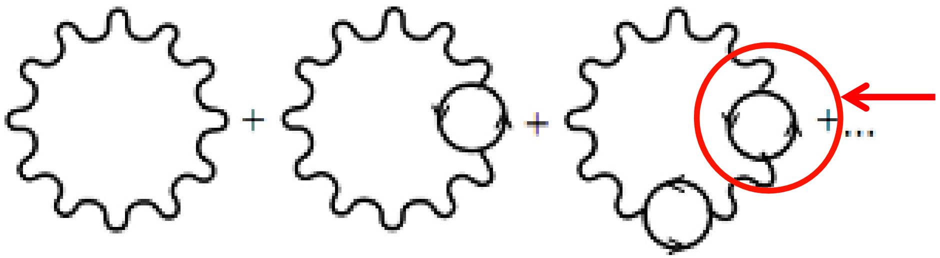
No odd powers of g !

QGP at large T

In the 80's: some attempts to perform systematic calculations for various fundamental quantities (pressure, damping rates,…)

Lot of confusions as well as gauge-dependent results

Solution: Late 80's, early 90's (Braaten & Pisarski):



Perturbation theory: 1 quark loop → g^2T^2

If gluon 4-momentum *k* is of the order gT, then each term is of the same order as the previous ones

$$\Delta + \Delta\Pi\Delta + \Delta\Pi\Delta\Pi\Delta + \cdots = g^2T^2 + g^2T^2 \times \frac{1}{g^2T^2} \times g^2T^2 + \cdots$$

=> need ressummation (leads to collective mode of mass ≈ gT).

So-called Hard Thermal Loop ressummation (Gauge invariant)

QGP at large T

In the 90's: systematic implementation of the HTL approach for the calculation of the pressure, up to order $g^6 \ln(g)$, in the "weak coupling" limit

P. B. Arnold and C.-X. Zhai, Phys. Rev. D 50 (1994), P. B. Arnold and C.-X. Zhai, Phys. Rev. D 51 (1995), E. Braaten and A. Nieto, Phys. Rev. Lett. 76 (1996) , E. Braaten and A. Nieto, Phys. Rev. D 53 (1996), C.-X. Zhai and B. M. Kastening, Phys. Rev. D 52 (1995) , K. Kajantie, M. Laine, K. Rummukainen and Y. Schroder Phys. Rev. D 67 (2003)

$$
\begin{aligned}
p = + \frac{8\pi^2 T^4}{45} \Bigg\{ & 1 + \frac{21}{32} N_f - \frac{15}{4}\left(1 + \frac{5}{12} N_f\right)\frac{\alpha_s}{\pi} + 30\left(1 + \frac{1}{6}N_f\right)^{3/2}\left(\frac{\alpha_s}{\pi}\right)^{3/2} \\
& + \Bigg[237.2 + 15.97 N_f - 0.413 N_f^2 + \frac{135}{2}\left(1 + \frac{1}{6}N_f\right)\log\left[\frac{\alpha}{\pi}\left(1 + \frac{1}{6}N_f\right)\right] \\
& \quad - \frac{165}{8}\left(1 + \frac{5}{12}N_f\right)\left(1 - \frac{2}{33}N_f\right)\log\frac{\mu}{2\pi T}\Bigg]\left(\frac{\alpha_s}{\pi}\right)^2 \\
& + \left(1 + \frac{1}{6}N_f\right)^{1/2}\Bigg[-799.2 - 21.96 N_f - 1.926 N_f^2 \\
& \quad + \frac{495}{2}\left(1 + \frac{1}{6}N_f\right)\left(1 - \frac{2}{33}N_f\right)\log\frac{\mu}{2\pi T}\Bigg]\left(\frac{\alpha_s}{\pi}\right)^{5/2} + \mathcal{O}\left(\alpha_s^3 \log \alpha_s\right)\Bigg\}
\end{aligned}
$$

μ: renormalization scale

QGP at large T

However, this series seems to be of "asymptotic" nature (converges just around g=0)

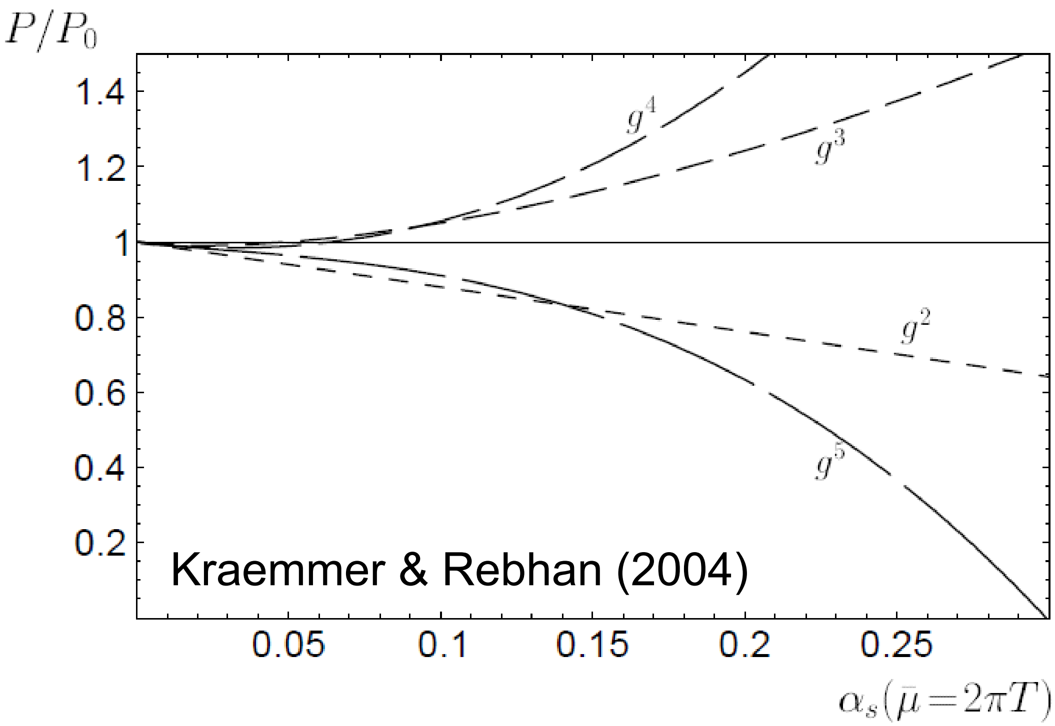
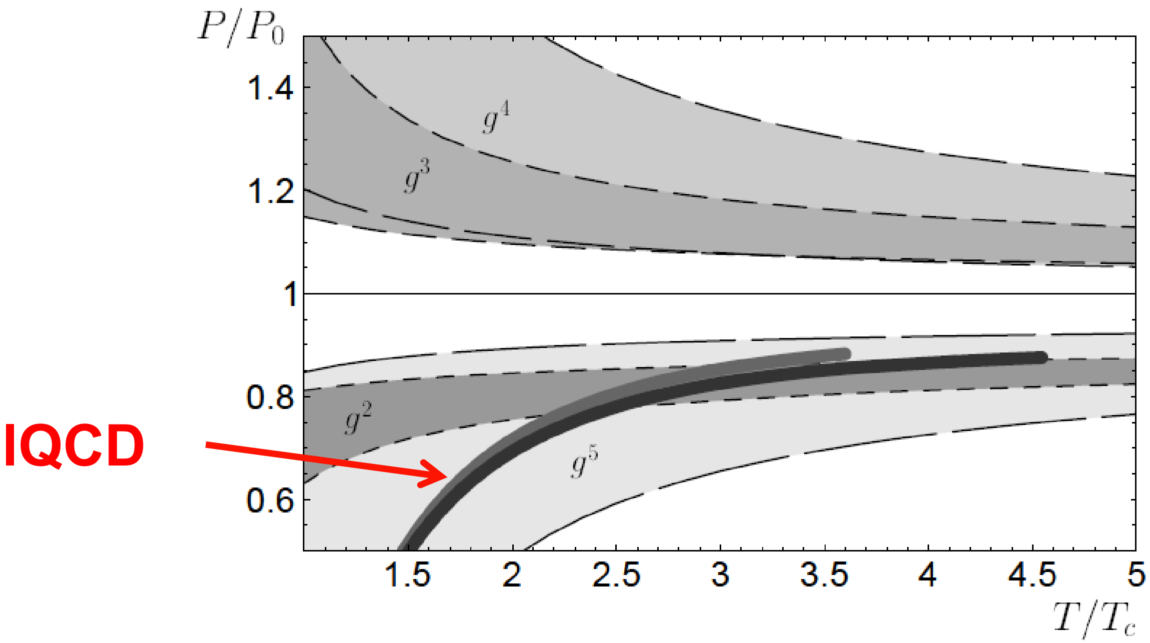


Figure 5. Strictly perturbative results for the thermal pressure of pure glue QCD normalized to the ideal-gas value as a function of $\alpha_s(\bar{\mu} = 2\pi T)$.

Additional problem: higher order terms are IR divergent (due to the unscreened magnetic modes in perturbative approaches)

QGP at large T

For values of the T achievable nowadays on earth, adding more and more terms simply leads to larger theoretical error bands !!!



Kraemmer & Rebhan (2004)

Figure 6. Strictly perturbative results for the thermal pressure of pure glue QCD as a function of T/T_c (assuming $T_c/\Lambda_{\overline{\rm MS}} = 1.14$). The various gray bands bounded by differently dashed lines show the perturbative results to order g^2, g^3, g^4, and g^5, using a 2-loop running coupling with $\overline{\rm MS}$ renormalization point $\bar{\mu}$ varied between πT and $4\pi T$. The thick dark-grey line shows the continuum-extrapolated lattice results from reference [154]; the lighter one behind that of a lattice calculation using an RG-improved action [155].

Need for further ressummations (early 2000's, fi: Blaizot, Iancu & Rebhan)

QGP at large T

An example of a recent work in HTL perturbation theory (J.O. Andersen et al, 2011)

Strategy (main lines):

✓ Add (and substract) the HTL Lagrangian

$$\mathcal{L} = \left(\mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm HTL}\right)\Big|_{g\to\sqrt{\delta}g} + \Delta\mathcal{L}_{\rm HTL}. \qquad \text{with}$$

$$\mathcal{L}_{\rm HTL} = -\frac{1}{2}(1-\delta)m_D^2 \,{\rm Tr}\left(G_{\mu\alpha}\left\langle \frac{y^\alpha y^\beta}{(y\cdot D)^2}\right\rangle_y G^\mu{}_\beta\right) + (1-\delta)\, i m_q^2 \bar{\psi}\gamma^\mu \left\langle \frac{y_\mu}{y\cdot D}\right\rangle_y \psi$$

HTLpt is defined by treating δ as a formal expansion parameter. By coupling the HTL improvement term (2.4) to the QCD Lagrangian (2.1), HTLpt systematically shifts the perturbative expansion from being around an ideal gas of massless particles which is the physical picture of the weak-coupling expansion, to being around a gas of massive quasi-particles which are the more appropriate physical degrees of freedom at high temperature.

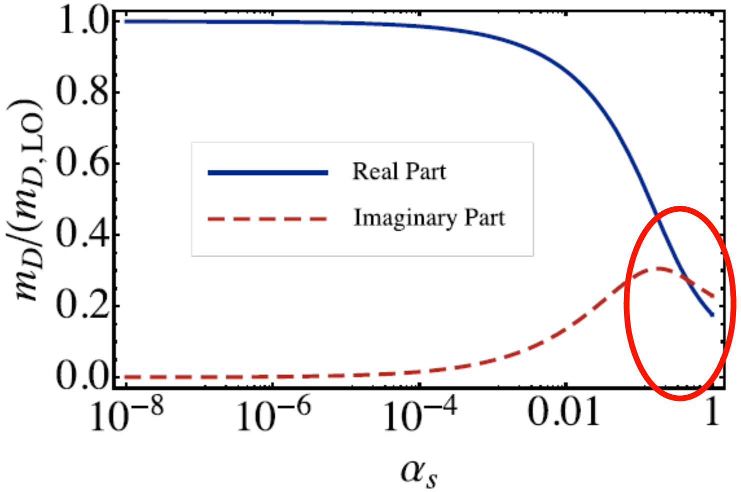
✓ Perform expansion wrt δ up to NNLO, and then set δ=1

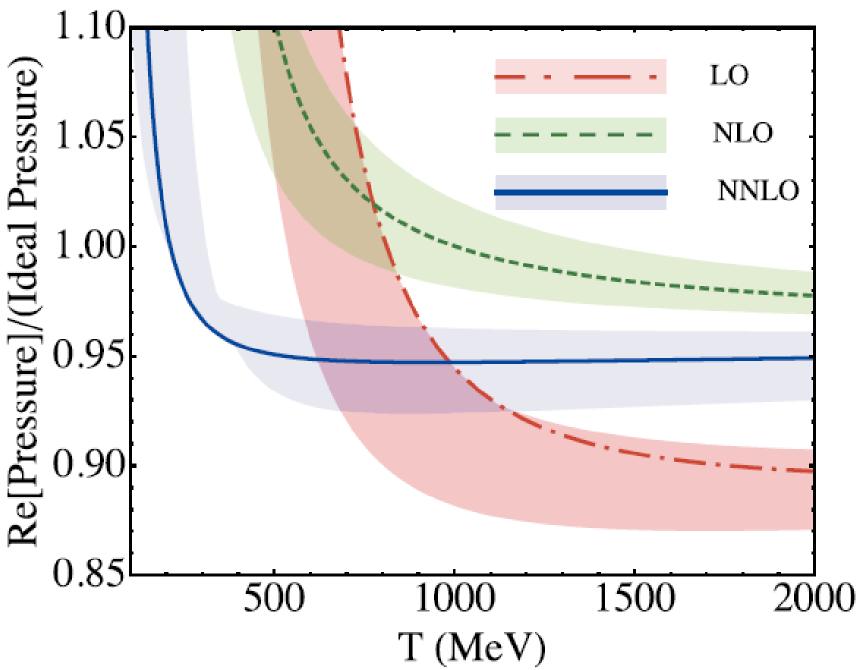
QGP at large T

✓ Adopt a prescription for *fixing* the Debye mass m_D and the dressed quark mass m_q:

a) Variational method

$$\begin{cases} \frac{\partial}{\partial m_D}\Omega(T,\alpha_s,m_D,m_q,\mu,\delta=1)=0 \\ \frac{\partial}{\partial m_q}\Omega(T,\alpha_s,m_D,m_q,\mu,\delta=1)=0 \end{cases}$$



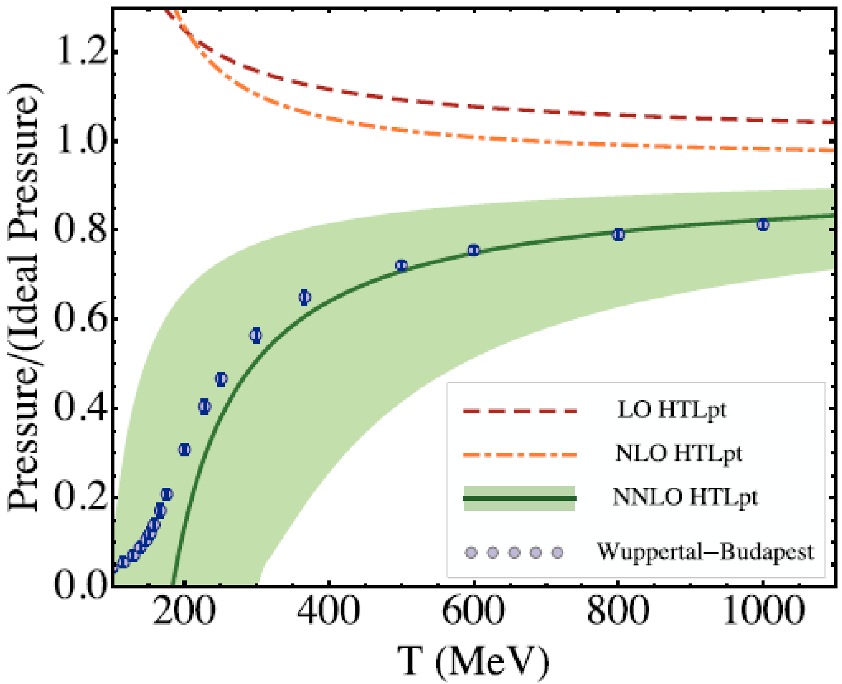


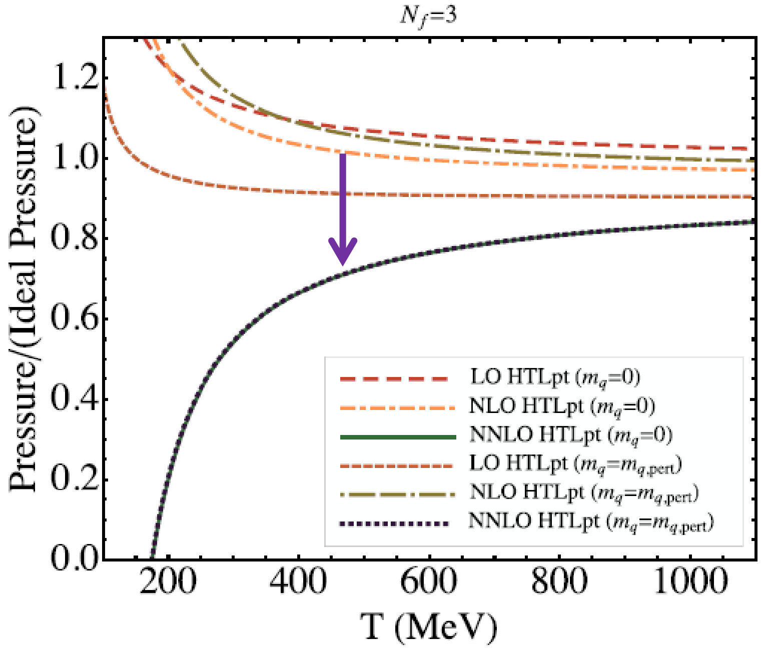
QGP at large T

✓ Adopt a prescription for *fixing* the Debye mass m_D and the dressed quark mass m_q:

b) m_D=m_E in Electrostatic QCD

Braaten & Nieto (1996)





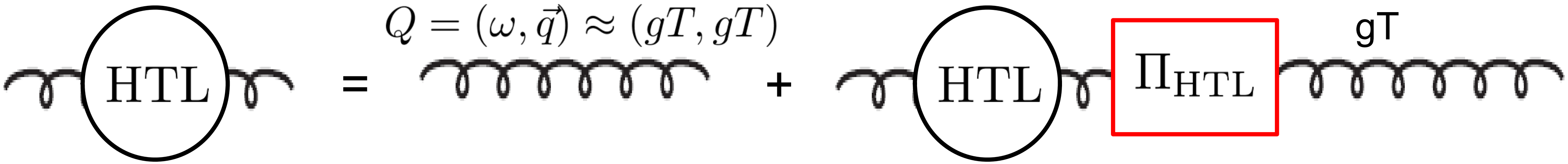
Significant NNLO contribution

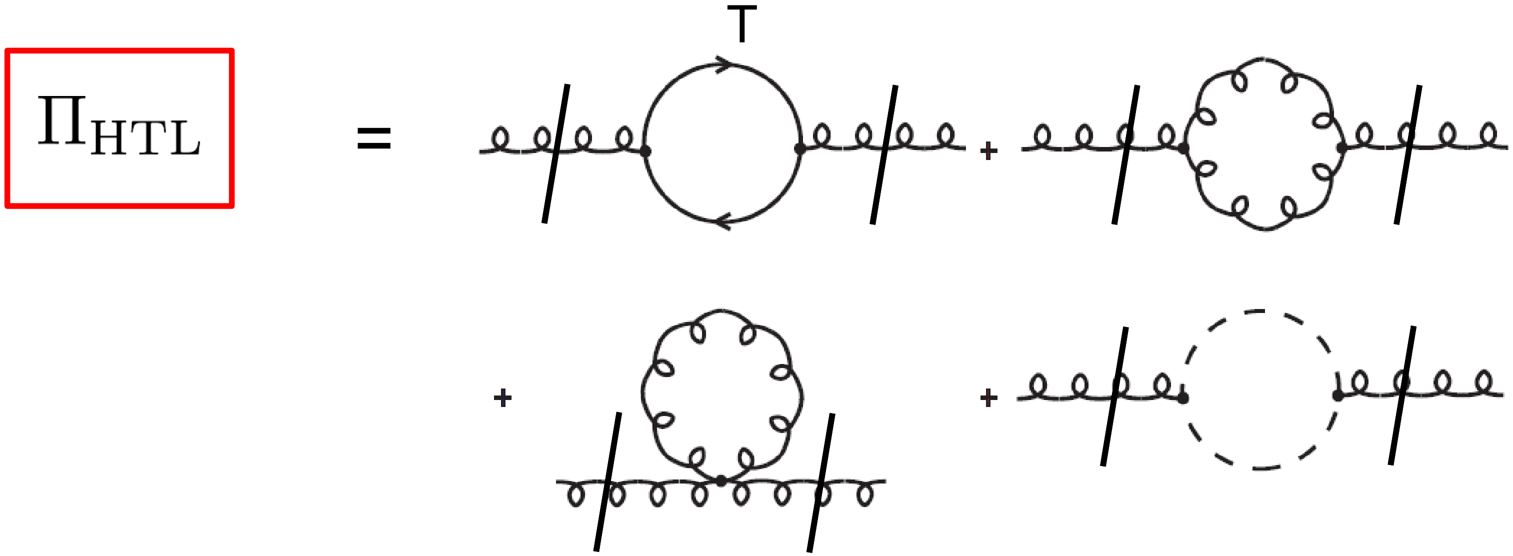
Chosen by the authors. "…That being said without lattice data to compare with one would be hard pressed to favor one prescription over the other "

Conclusion: possibility to describe lQCD for T>Tc with quasi-particle approach !!!

QGP at large T: quasi-particles

<u>soft modes in real-time PT:</u> HTL resummation for the gluon propagator:



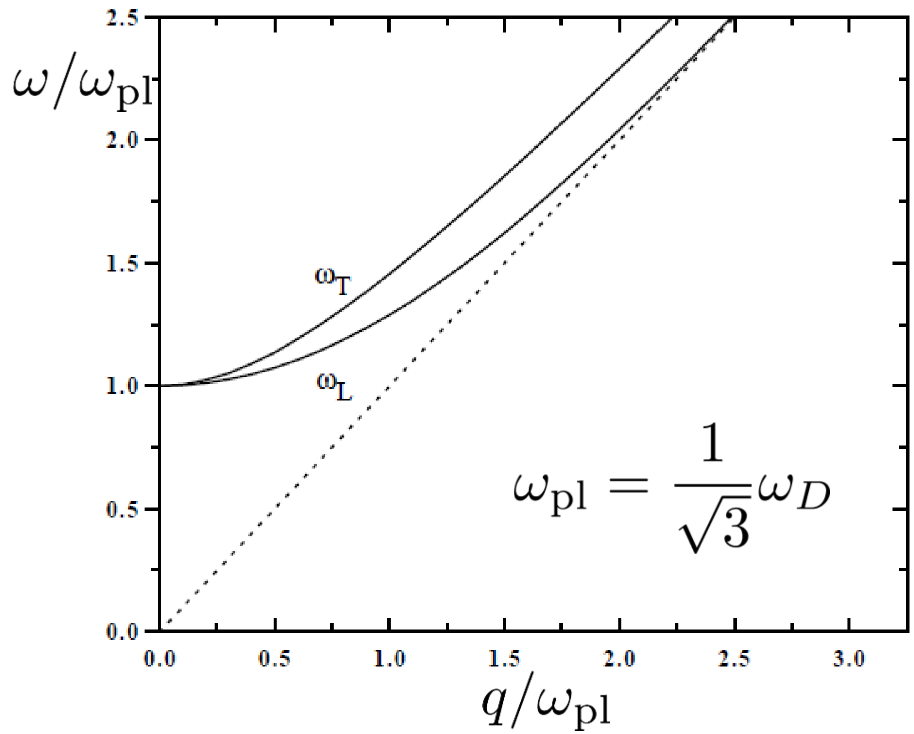


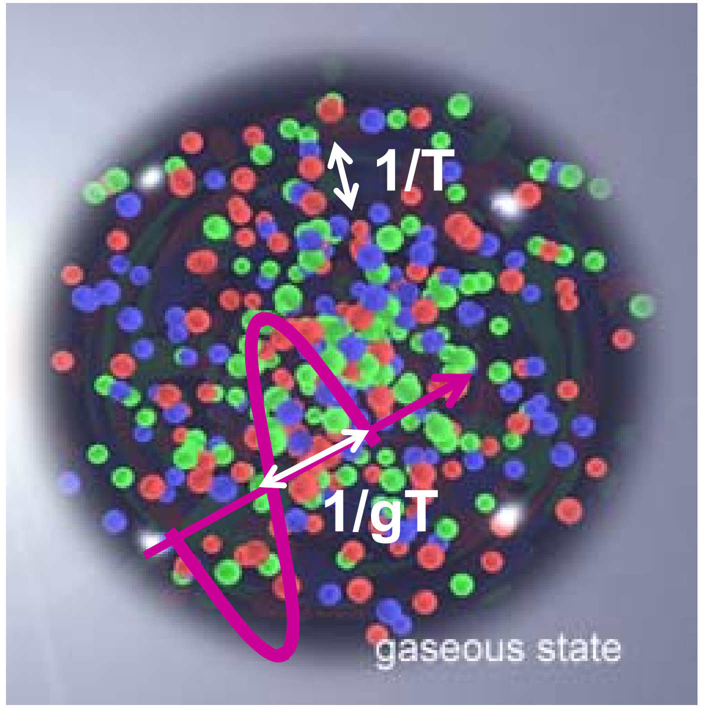
$$\begin{cases} \Pi_L(Q) = -\omega_D^2 \dfrac{Q^2}{q^2}\left(1 - F(\omega/q)\right) \\ \Pi_T(Q) = \dfrac{\omega_D^2}{2}\left[1 + \dfrac{Q^2}{q^2}\left(1 - F(\omega/q)\right)\right] \end{cases} \quad \text{with} \quad \begin{cases} F(x) = \dfrac{x}{2}\left[\ln\left|\dfrac{x+1}{x-1}\right| - i\pi\theta(1-|x|)\right] \\ \omega_D^2 = \dfrac{1}{3}g^2T^2\left(N_c + \dfrac{1}{2}N_f\right) \end{cases}$$

QGP at large T: quasi-particles

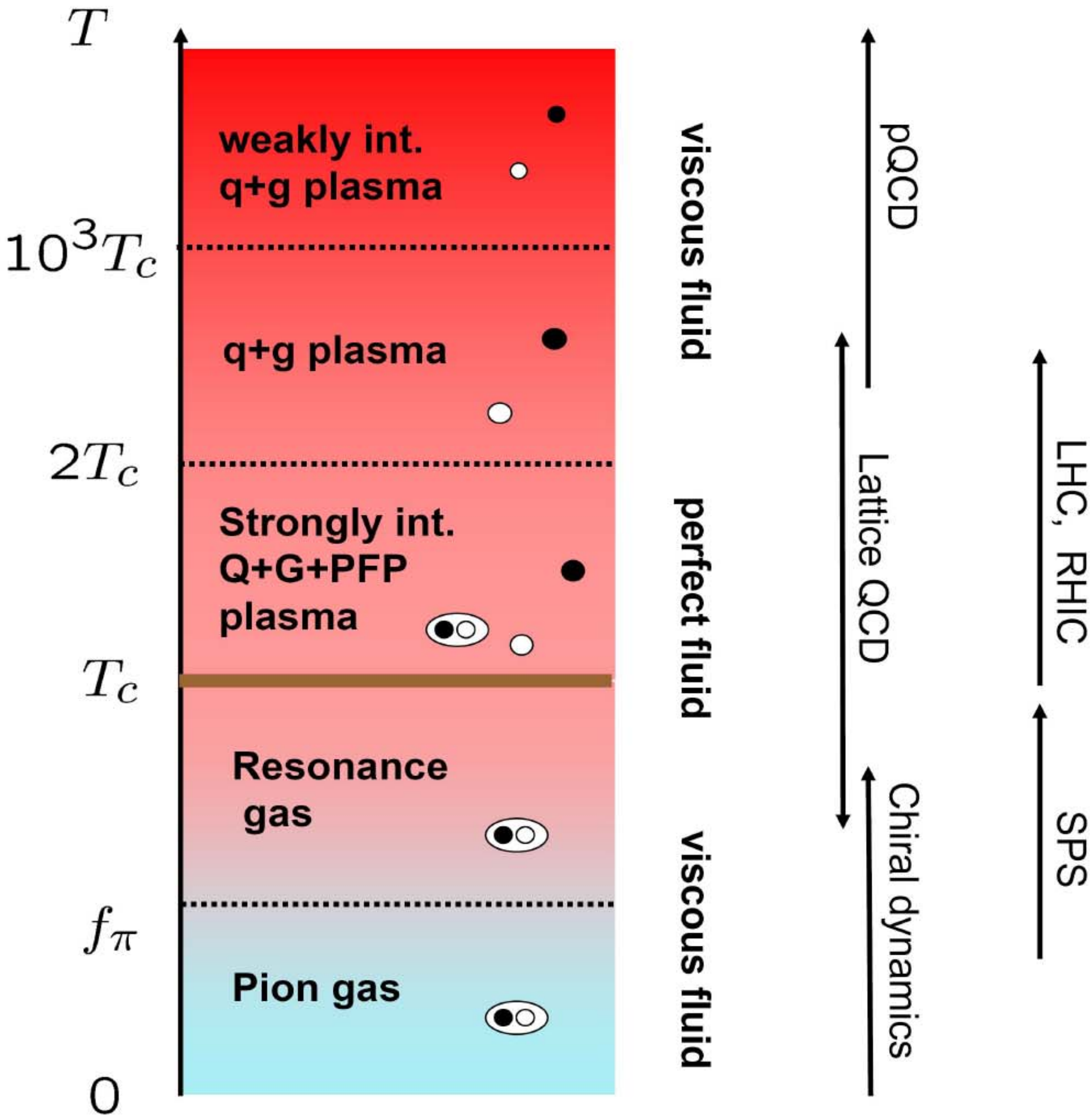
Several interesting features:

- ➢ Debye screening
- ➢ Landau Damping (for space-like propagation)
- ➢ Collective modes (plasmons): poles of the HTL propagator $Q^2 - \Pi_{L/T}(Q) = 0$

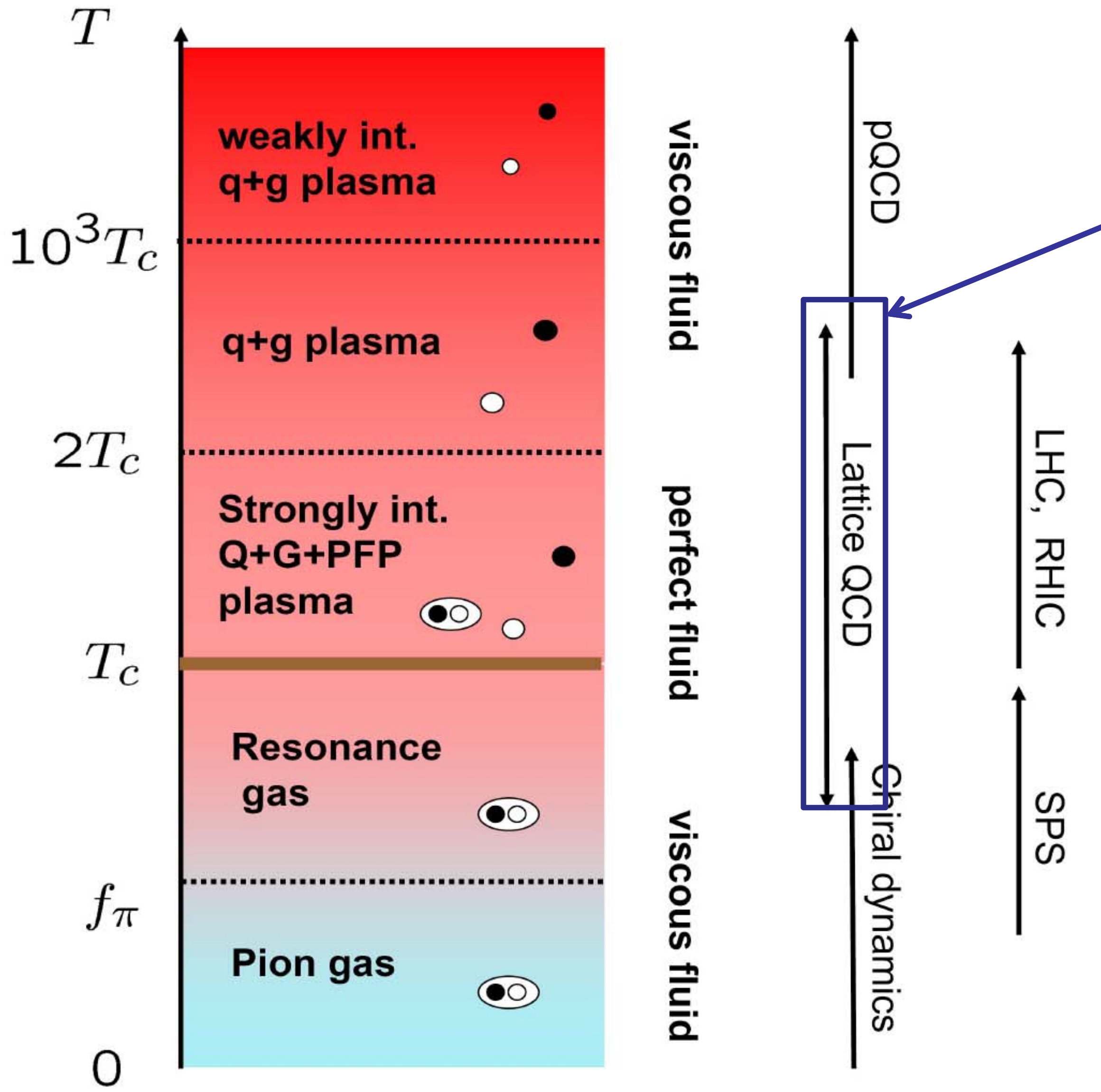




A less simplistic picture of the QG "P"



A less simplistic picture of the QG "P"



Slow progress (no talk on viscosity in lattice 2012, lattice 2013) + « No go » for t-evolving systems

Models

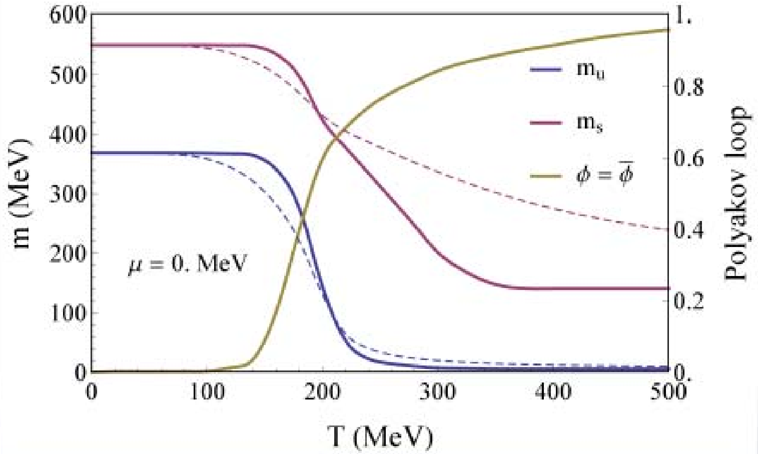
Models of QCD at finite T: Polyakov-NJL

From R. Marty, SQM 2013 (Nantes-Frankfurt collab.)

Lagrangian:

$$\mathscr{L}_{PNJL} = \bar\psi\,(i\not{D} - m_0)\,\psi + \mathcal{U}(\phi,\bar\phi,T) + \mu\bar\psi\gamma_0\psi$$
$$+\ G\sum_{a=0}^{8}\left[(\bar\psi\lambda^a\psi)^2 + (\bar\psi i\gamma_5\lambda^a\psi)^2\right]$$
$$+\ K\left[\det\bar\psi\,(1-\gamma_5)\,\psi + \det\bar\psi\,(1+\gamma_5)\,\psi\right]$$

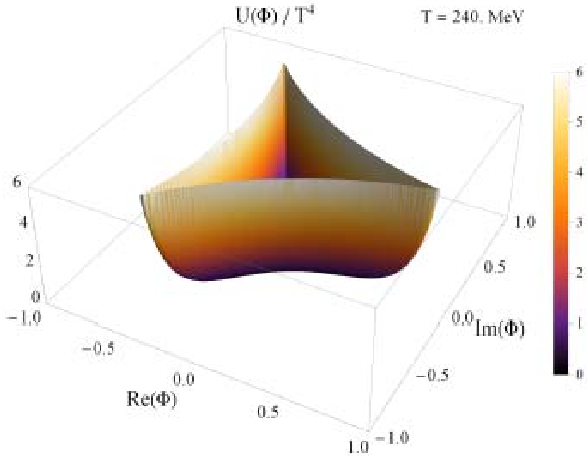
No gluon as dynamical dof, glue fields in the P-loop (Φ)



Based on PRD 79, 116003 (2009)

Modified dist.:

$$f_q \to f_q^{\Phi}(\mathbf{p}, T, \mu)$$
$$f_{\bar q} \to f_{\bar q}^{\Phi}(\mathbf{p}, T, \mu)$$

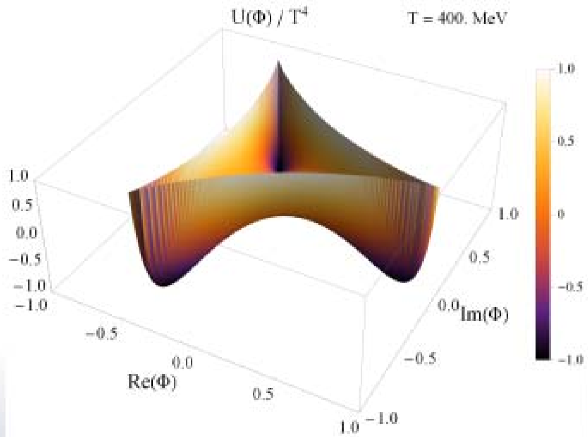


Modified chiral cond.:

$$\langle\langle\bar\psi\psi\rangle\rangle = -2N_c\int_0^{\Lambda}\frac{d^3p}{(2\pi)^3}\frac{m}{E_p}\left[1 - f_q^{\Phi} - f_{\bar q}^{\Phi}\right]$$

Effective potential:

$$\frac{\mathcal{U}(\Phi,\bar\Phi,T)}{T^4} = -\frac{a(T)}{2}\bar\Phi\Phi + b(T)$$
$$\log[1 - 6\bar\Phi\Phi + 4(\bar\Phi^3 + \Phi^3) + 3(\bar\Phi\Phi)^2]$$



Models of QCD at finite T: Dynamical Quasi-Particle

From R. Marty, SQM 2013 — Idea from Peshier & Cassing (2000 's)

Quasi-partons:

Masses:

$$M_g^2(T,\mu_q) = \frac{g^2}{6}\left(\left(N_c + \frac{N_f}{2}\right)T^2 + \frac{N_c}{2}\sum_q \frac{\mu_q^2}{\pi^2}\right)$$

$$M_{q/\bar{q}}^2(T,\mu_q) = \frac{N_c^2-1}{8N_c} g^2 \left(T^2 + \sum_q \frac{\mu_q^2}{\pi^2}\right)$$

Widths:

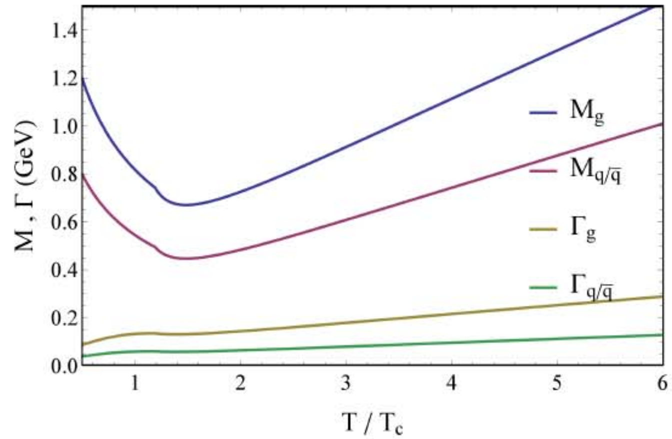
$$\Gamma_g(T) = \frac{1}{3} N_c \frac{g^2 T}{8\pi} \ln\left(\frac{2c}{g^2}+1\right),$$

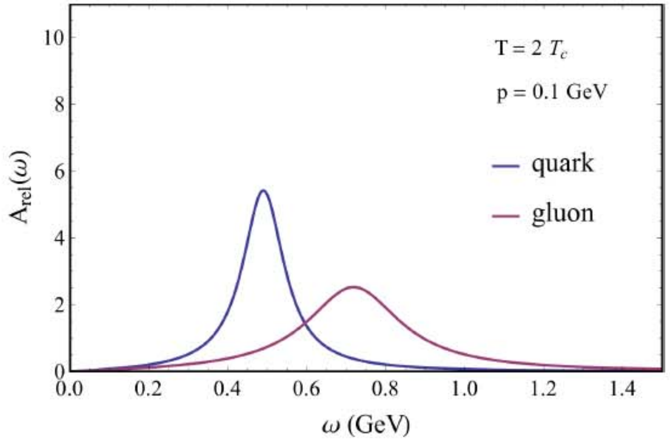
$$\Gamma_{q/\bar{q}}(T) = \frac{1}{3}\frac{N_c^2-1}{2N_c}\frac{g^2 T}{8\pi} \ln\left(\frac{2c}{g^2}+1\right),$$

Coupling constant:

$$g^2(T/T_c) = \frac{48\pi^2}{(11N_c - 2N_f)\ln[\lambda^2(T/T_c - T_s/T_c)^2]}$$

Based on EPJ ST 168, 3 (2009)





Off-shellness:

Breit-Wigner spectral function:

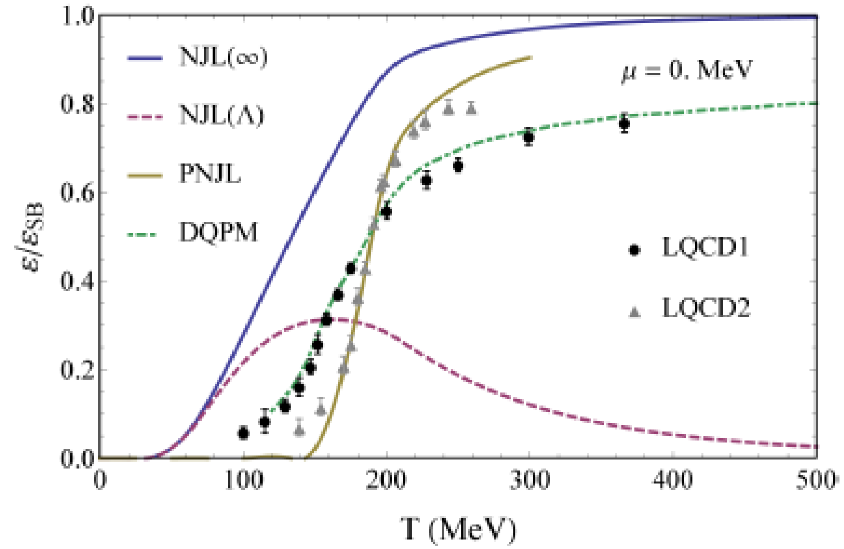
$$A(\omega,\mathbf{p}) = \frac{\Gamma}{E}\left(\frac{1}{(\omega-E)^2+\Gamma^2} - \frac{1}{(\omega+E)^2+\Gamma^2}\right)$$

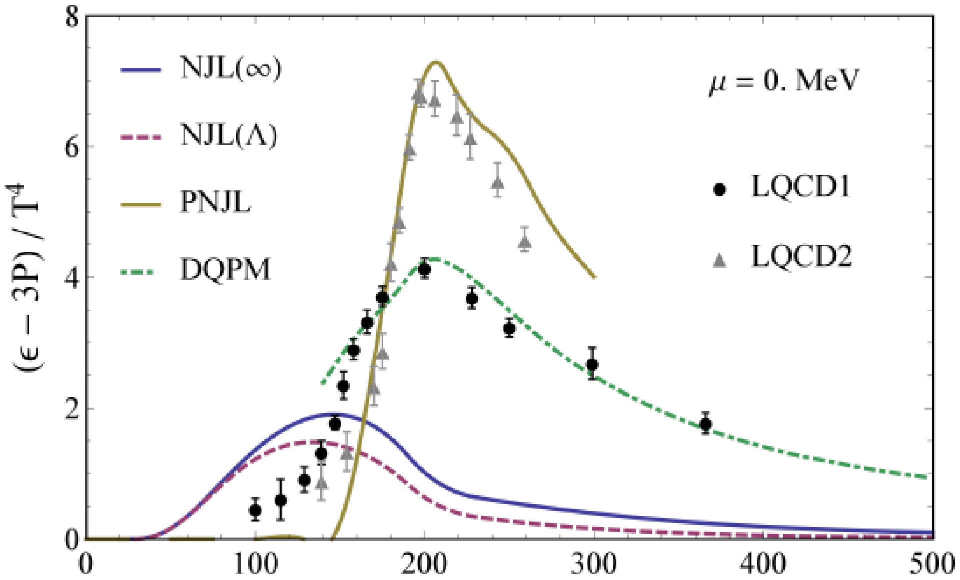
with $E^2 = \mathbf{p}^2 + M^2 - \Gamma^2$ and

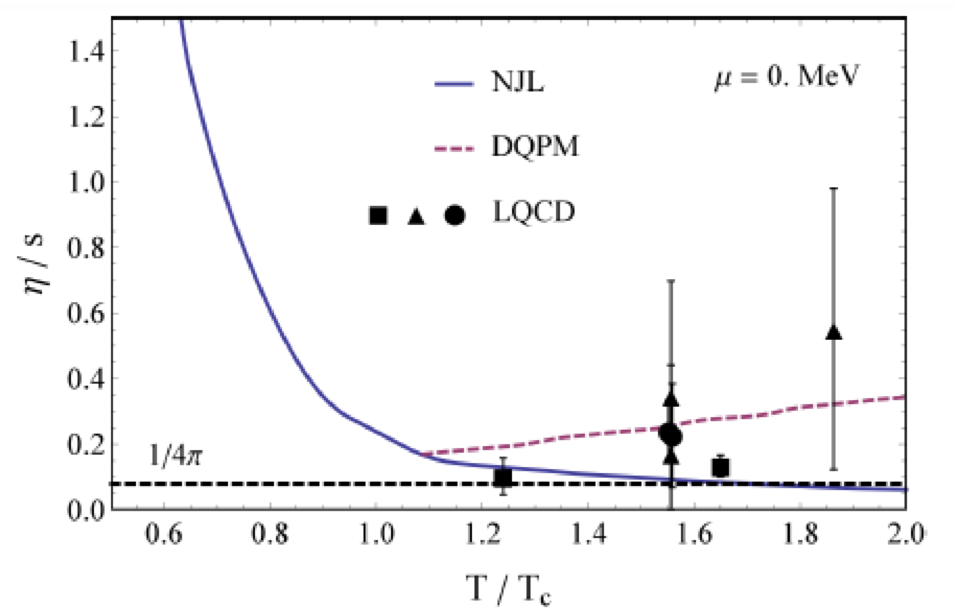
$$\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\,\omega A(\omega,\mathbf{p}) = \int_0^{\infty}\frac{d\omega}{2\pi}\,2\omega A(\omega,\mathbf{p}) = 1$$

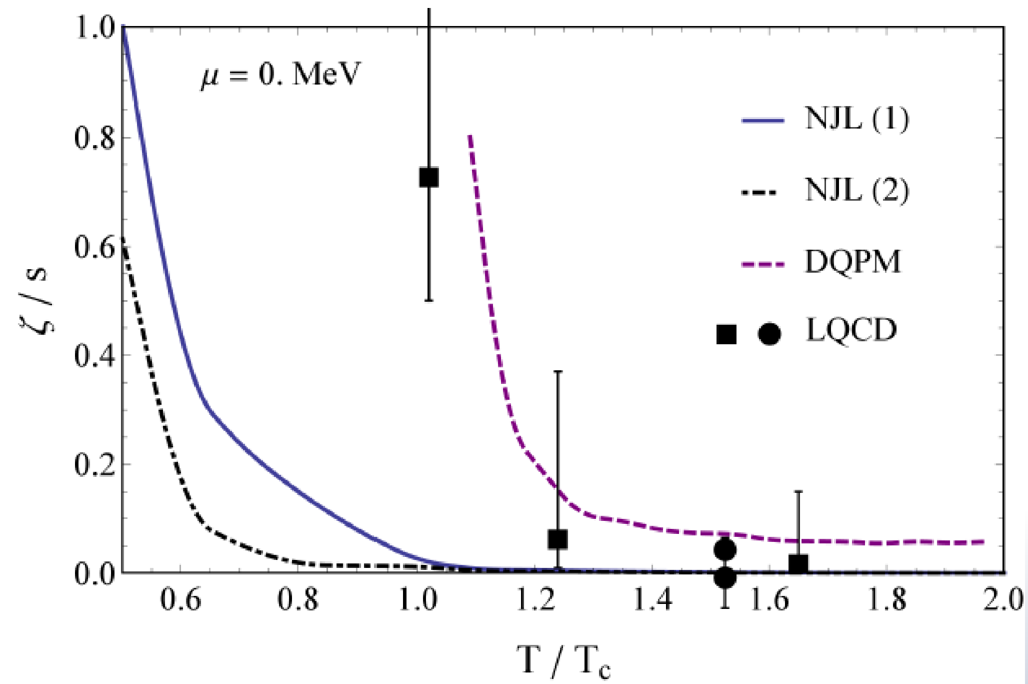
Models of QCD at finite T: some results

From R. Marty, SQM 2013

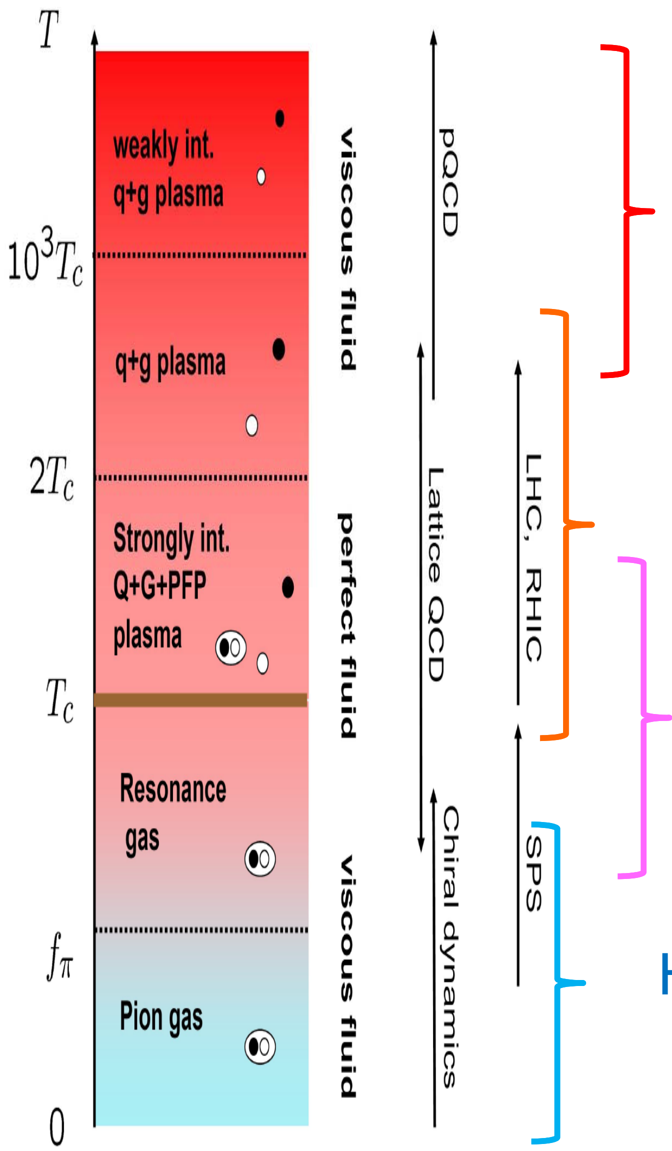








Models of URHIC



BAMPS: Partonic transport using 2↔2 and 2↔3 cross sections

NJL & pHSD: transport approaches based on effective dof

Fluid dynamics with various initial conditions, EOS, freeze out prescriptions

HSD, URQMD: Hadronic cascades

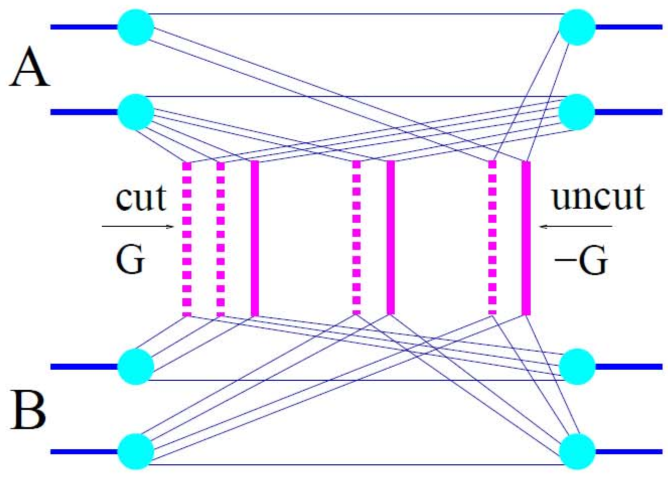
Models of URHIC: The EPOS approach

SQM2013, Birmingham, July 2013 - Klaus Werner - Subatech, Nantes (

EPOS: Marriage pQCD+GRT+energy sharing

(Drescher, Hladik, Ostapchenko, Pierog, and Werner, Phys. Rept. 350, 2001)

$$\sigma^{\mathrm{tot}} = \sum_{\mathrm{cut\,P}} \int \sum_{\mathrm{uncut\,P}} \int \underbrace{\qquad\qquad}_{d\sigma_{\mathrm{exclusive}}}$$



$d\sigma_{\mathrm{exclusive}}$

Another aspect of nuclei

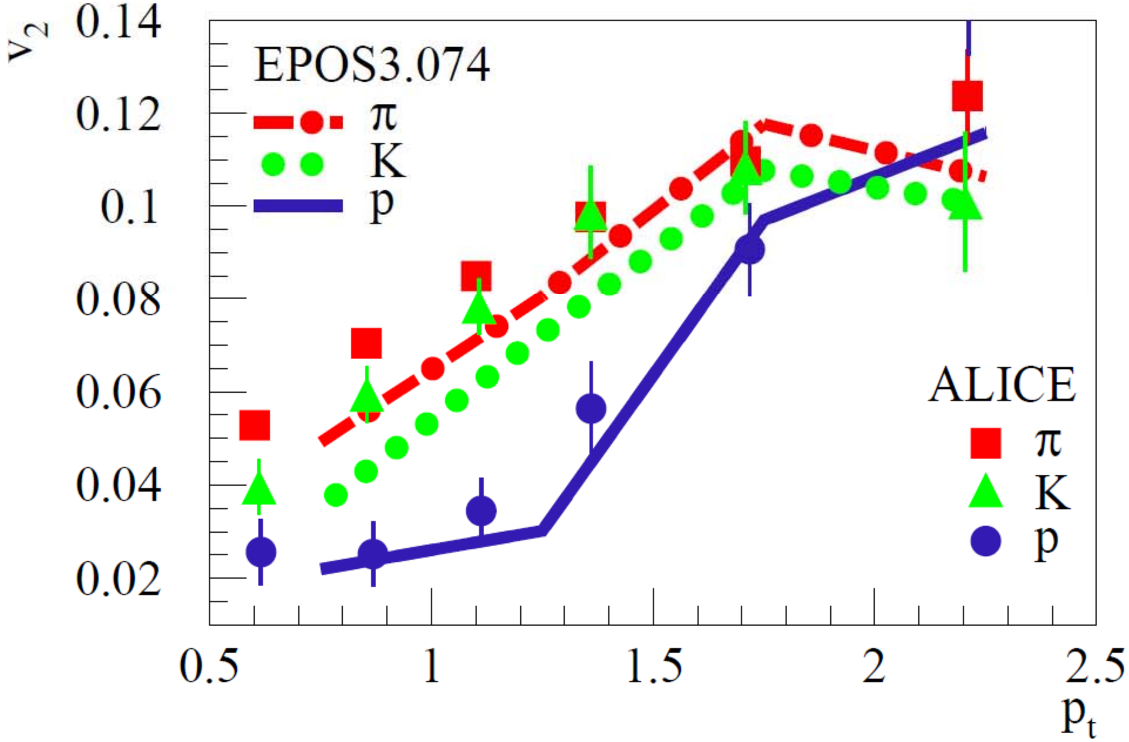
$$\mathrm{cut\,Pom}: G = \frac{1}{2\hat{s}} 2\mathrm{Im}\,\{\mathcal{FT}\{T\}\}(\hat{s}, b),\; T = i\hat{s}\,\sigma_{hard}(\hat{s})\,\exp(R^2_{\mathrm{hard}}t)$$

Nonlinear effects considered via saturation scale $Q_s \propto N_{part}\,\hat{s}^{\lambda}$

Models of URHIC: The EPOS approach

RANP 2013 Rio de Janeiro – Klaus Werner – Subatech, Nantes 0-34

v2 for π, K, p clearly differ



QGP In p-Pb ?!

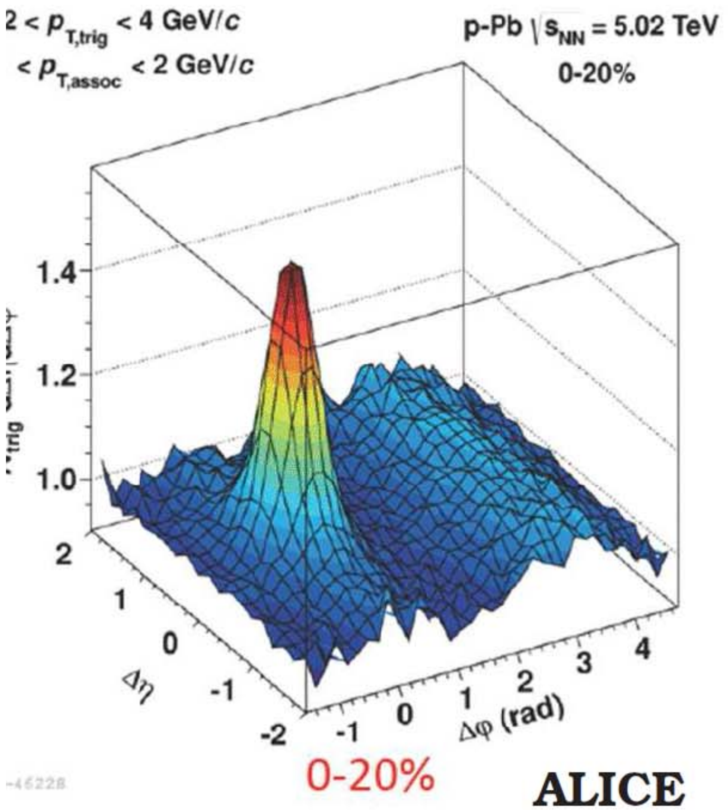
mass splitting, due to flow

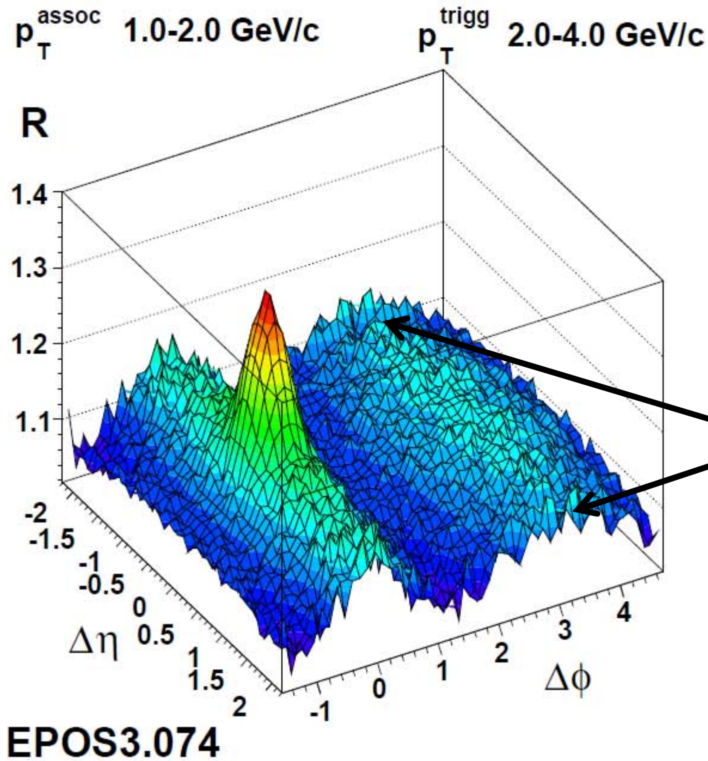
Models of URHIC: The EPOS approach

RANP 2013 Rio de Janeiro – Klaus Werner – Subatech, Nantes 0·

"Ridges" in pA

ALICE, arXiv:1212.2001, arXiv:1307.3237





Rapidity correlations as a reminiscence of the initial state

Conclusions and Perspectives

- Lively and ongoing debate on the interpretation of the hot and dense matter formed in URHIC and its various avatars
- Even more interesting: is there a "simple" (quasi-particle like, AdS/CFT,…) way to understand the physical observables around T_c-$2T_c$?
- Need for further developments in lQCD and HTP-pT for more observables
- (I tried to convince you that) QGP physics is both: nuclear physics, particle physics, hadronic physics, statistical physics, mathematical physics, still open for major discoveries…