Formation of Super-Heavy Elements

— Uncertainties in theoretical modeling

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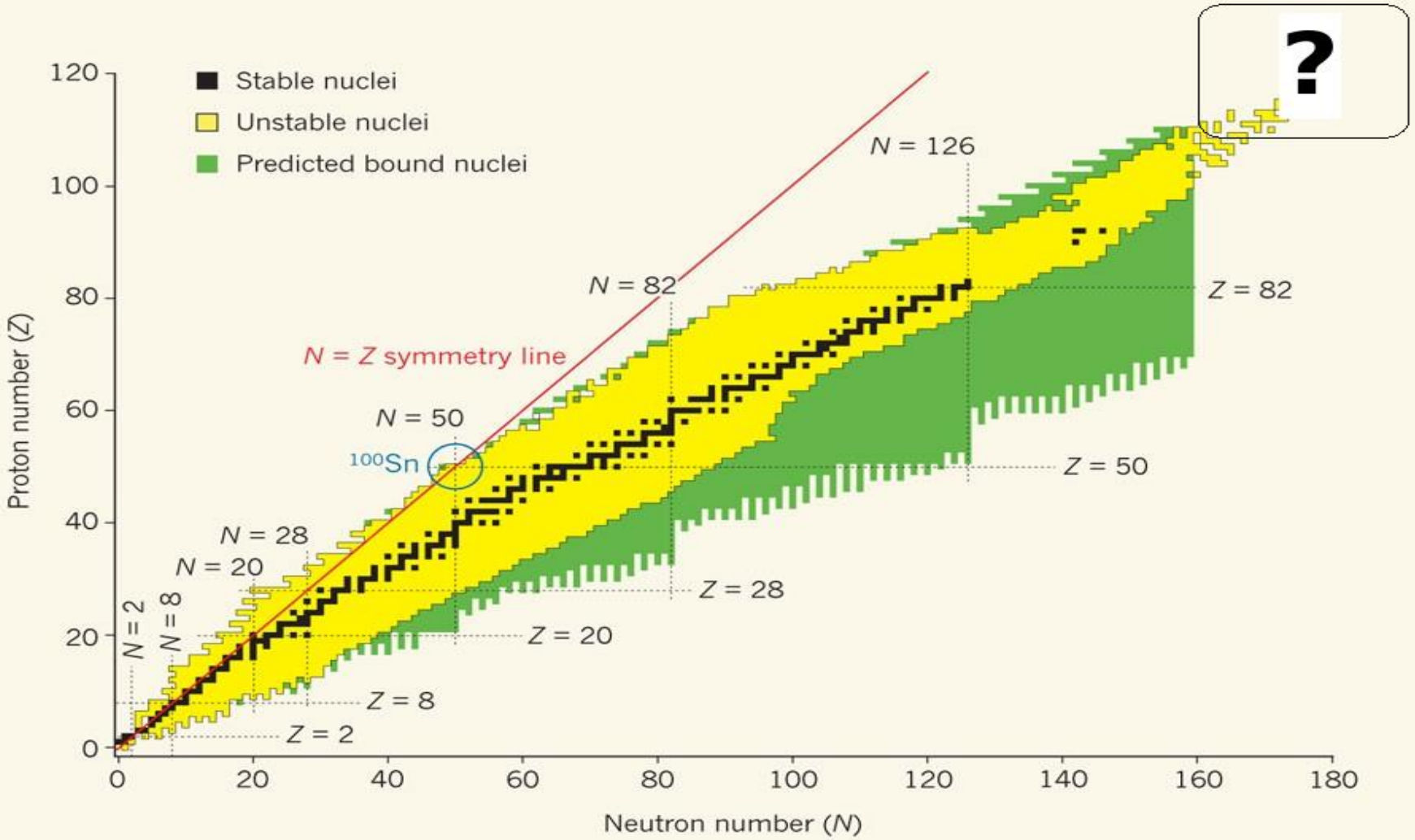
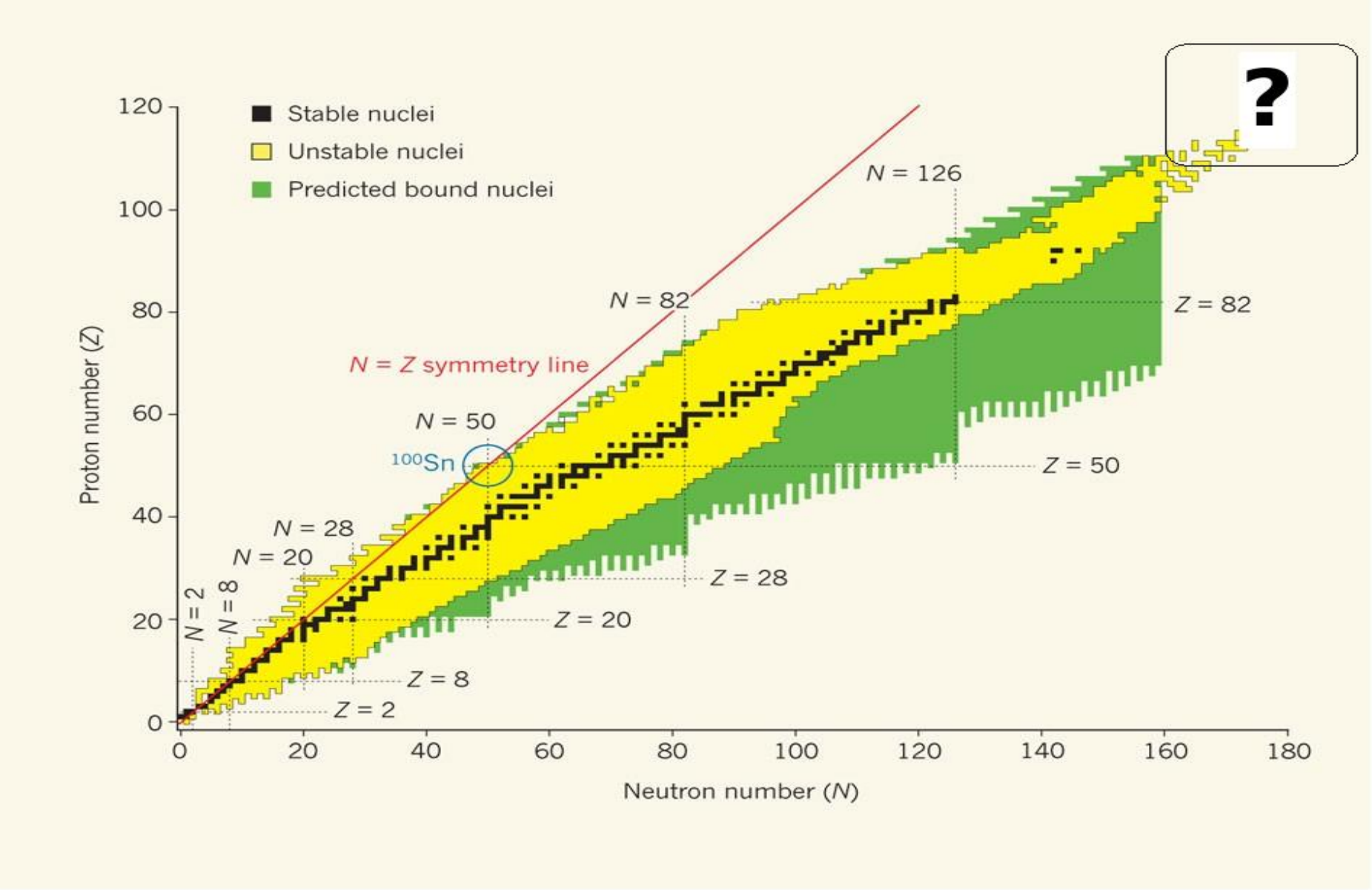
December 16, 2014

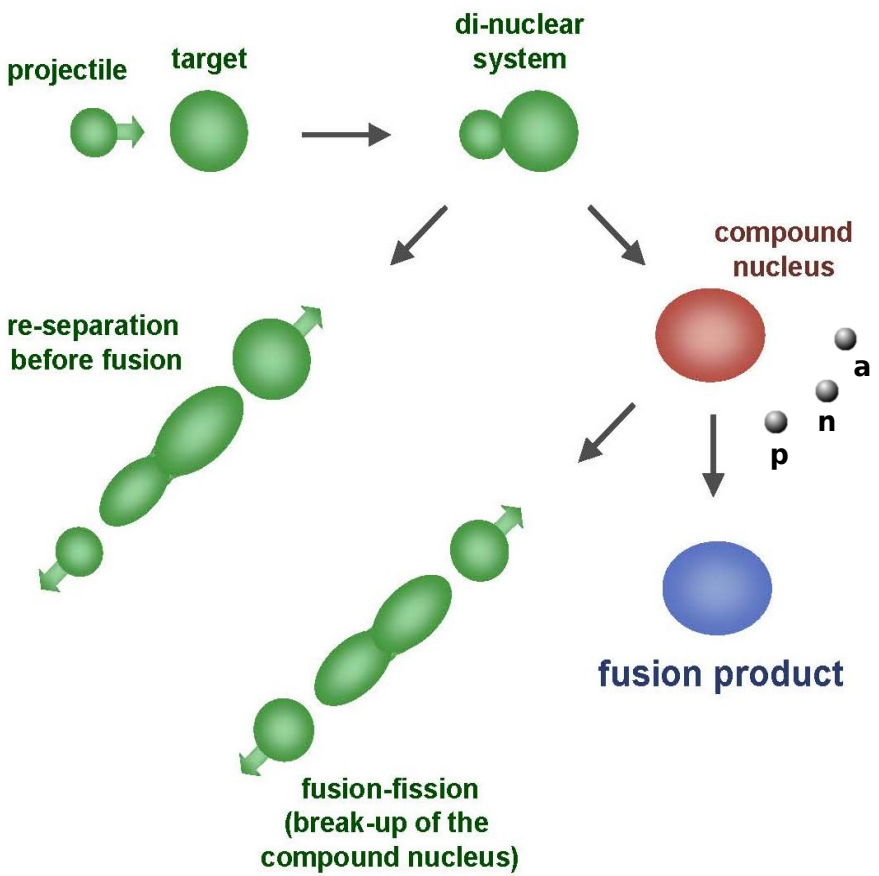

?
Stable nuclei
Unstable nuclei
Predicted bound nuclei
N = 126
N = 82
Z = 82
N = Z symmetry line
N = 50
100Sn
Z = 50
N = 28
N = 20
N = 2
N = 8
Z = 28
Z = 20
Z = 8
Z = 2
Proton number (Z)
Neutron number (N)
0
20
40
60
80
100
120
140
160
180



Where is the “Island of Stability”? How to get there?

Schematic of fusion-evaporation reaction

Fusion-evaporation reaction:



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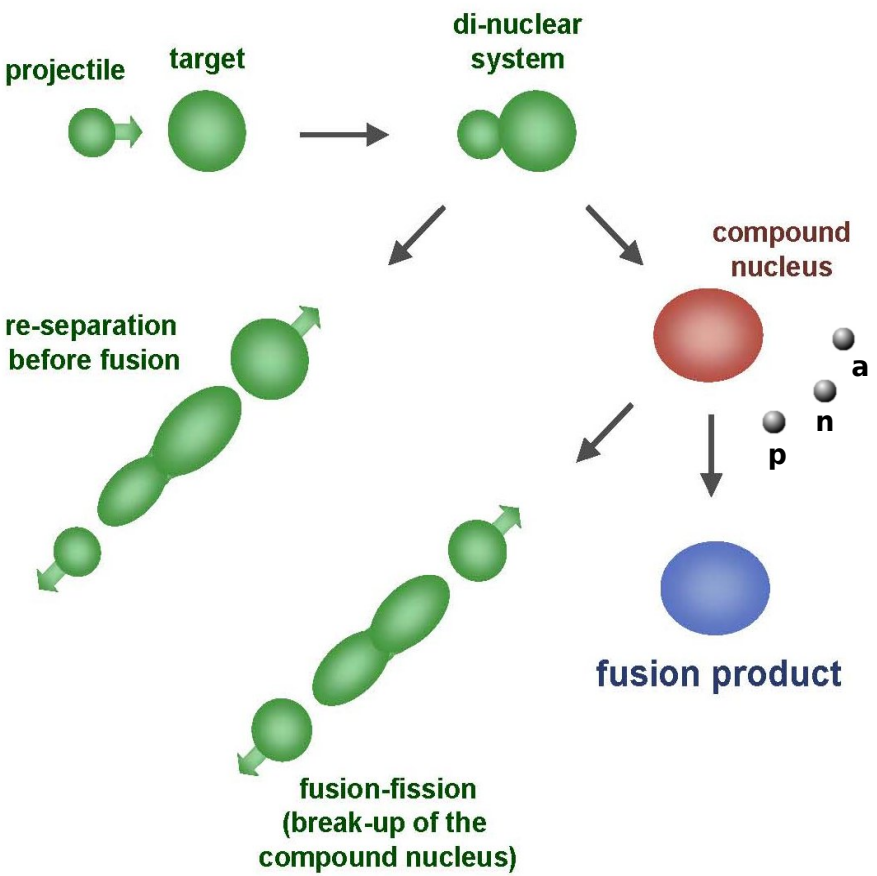

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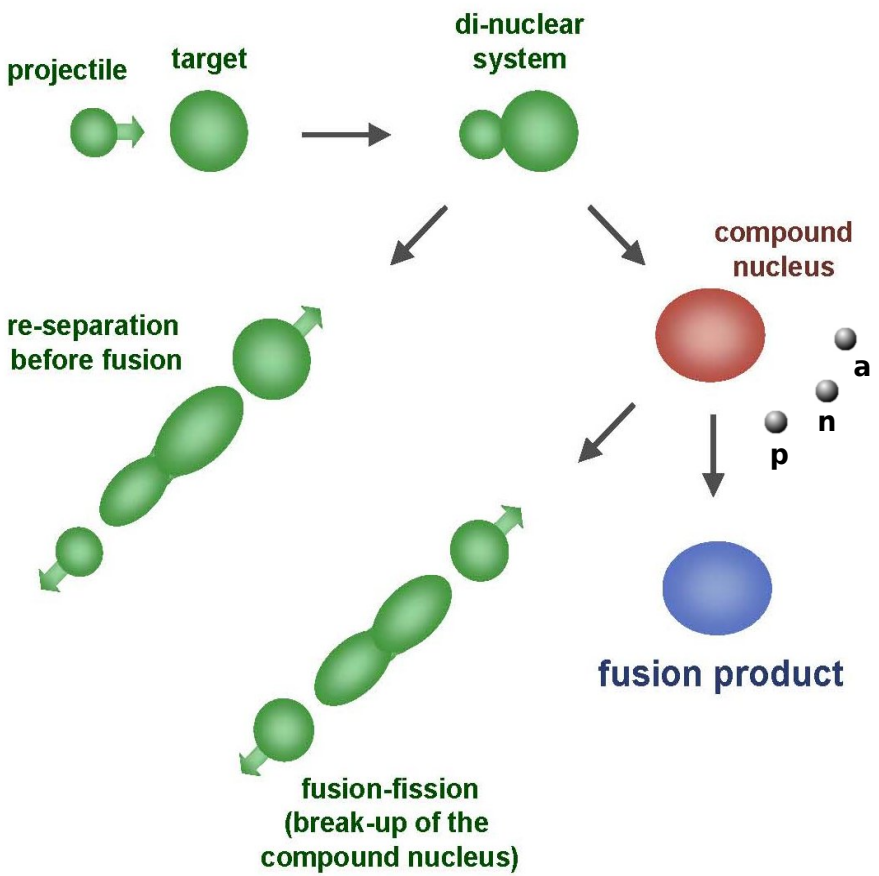

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... Re-separation (or quasi-fission) process only occurs in heavy binary systems. This phenomenon is usually called the "Fusion hindrance". Key open question for the synthesis of SHE!

Evaporation-residue (ER) cross-setion of SHE

$$\sigma_{\mathrm{res}} = \sigma_{\mathrm{cap}} \times P_{\mathrm{form}} \times W_{\mathrm{sur}}$$

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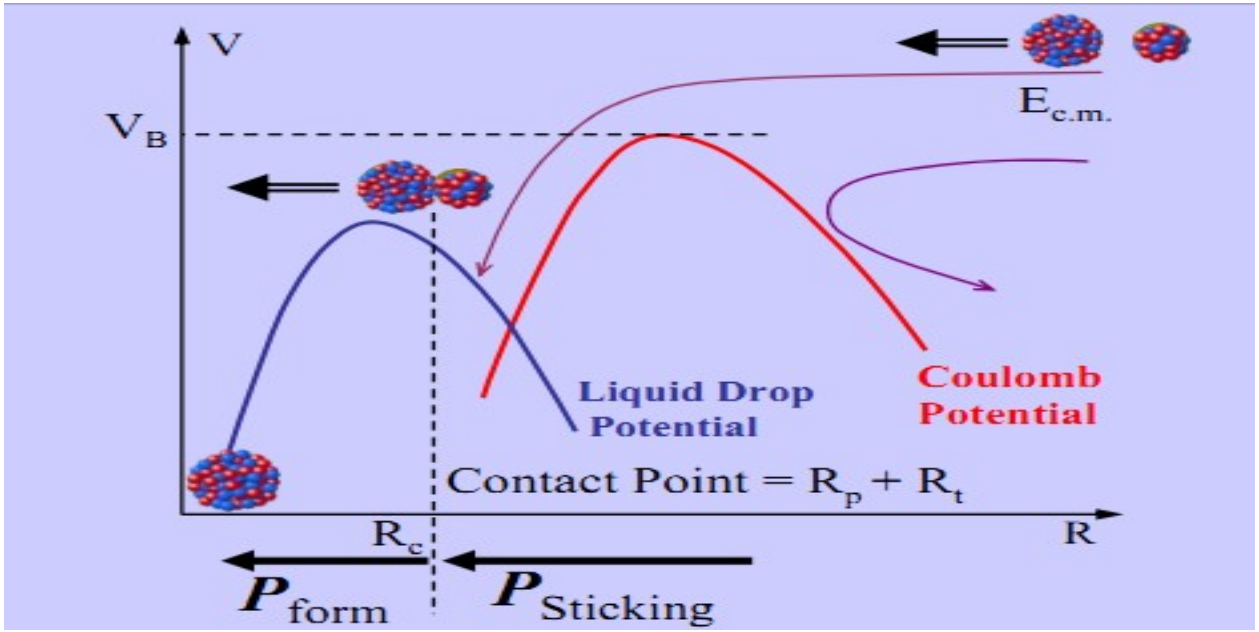
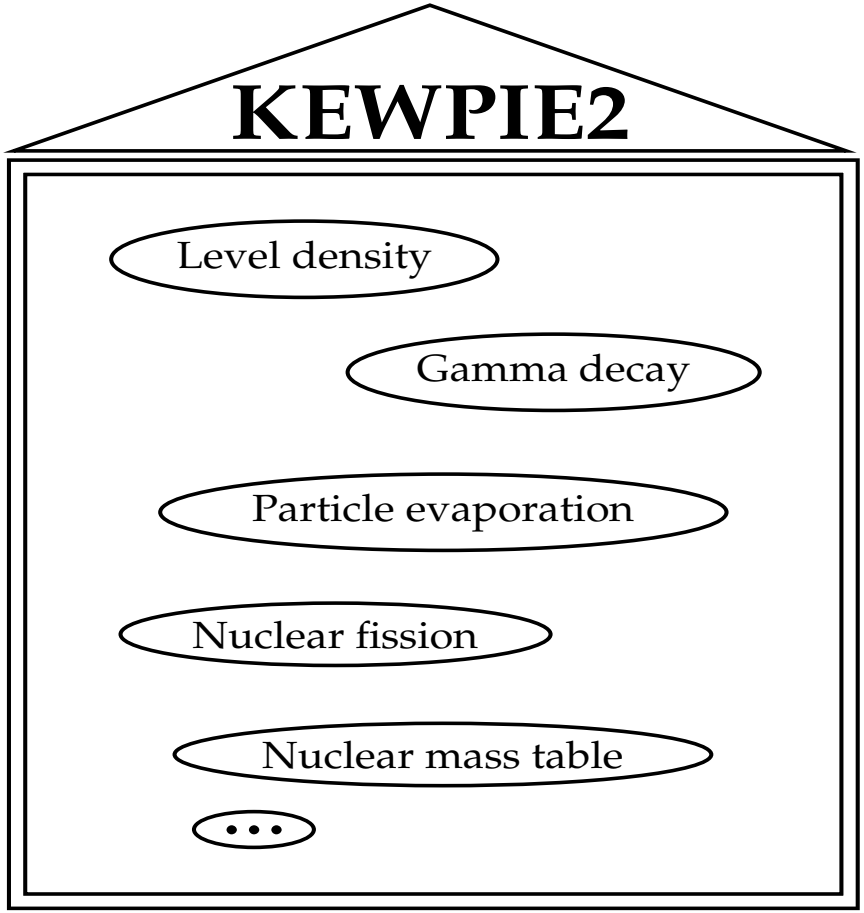


For light systems, $P_{\rm form} = 1$;

For heavy systems ($Z_t \cdot Z_p \gtrsim 1600 - 1800$), $P_{\rm form} < 1$.

W_{sur}? KEWPIE2 code ... For guiding experiments



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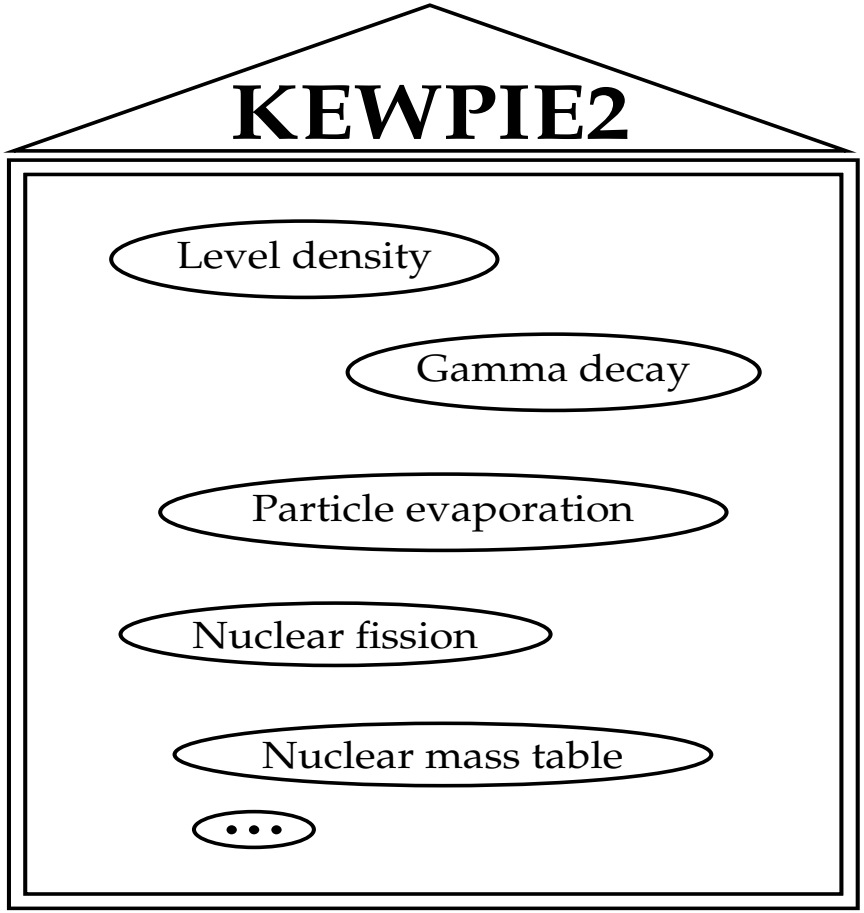

Basic features:

- Not a Monte-Carlo cascade code, due to low survival probabilities.
- Based on the discretization of the energy spectrum. More efficient!

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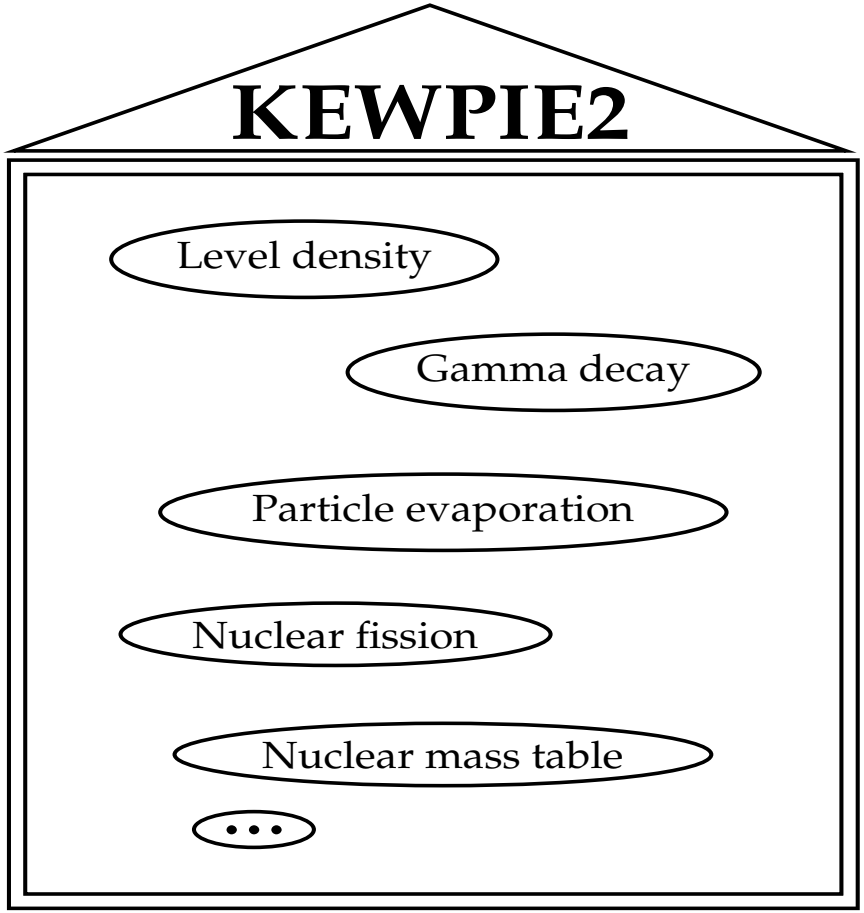

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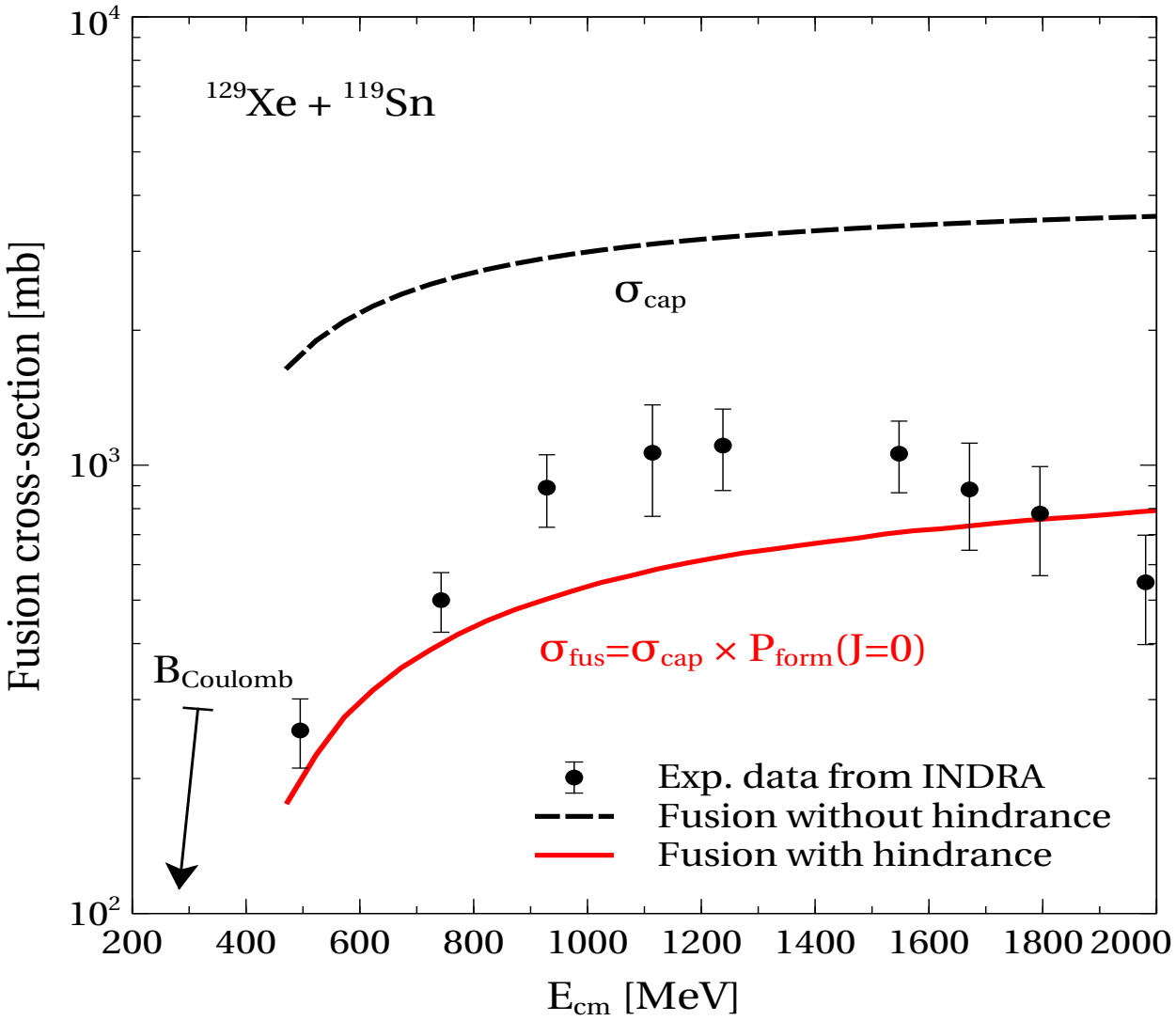
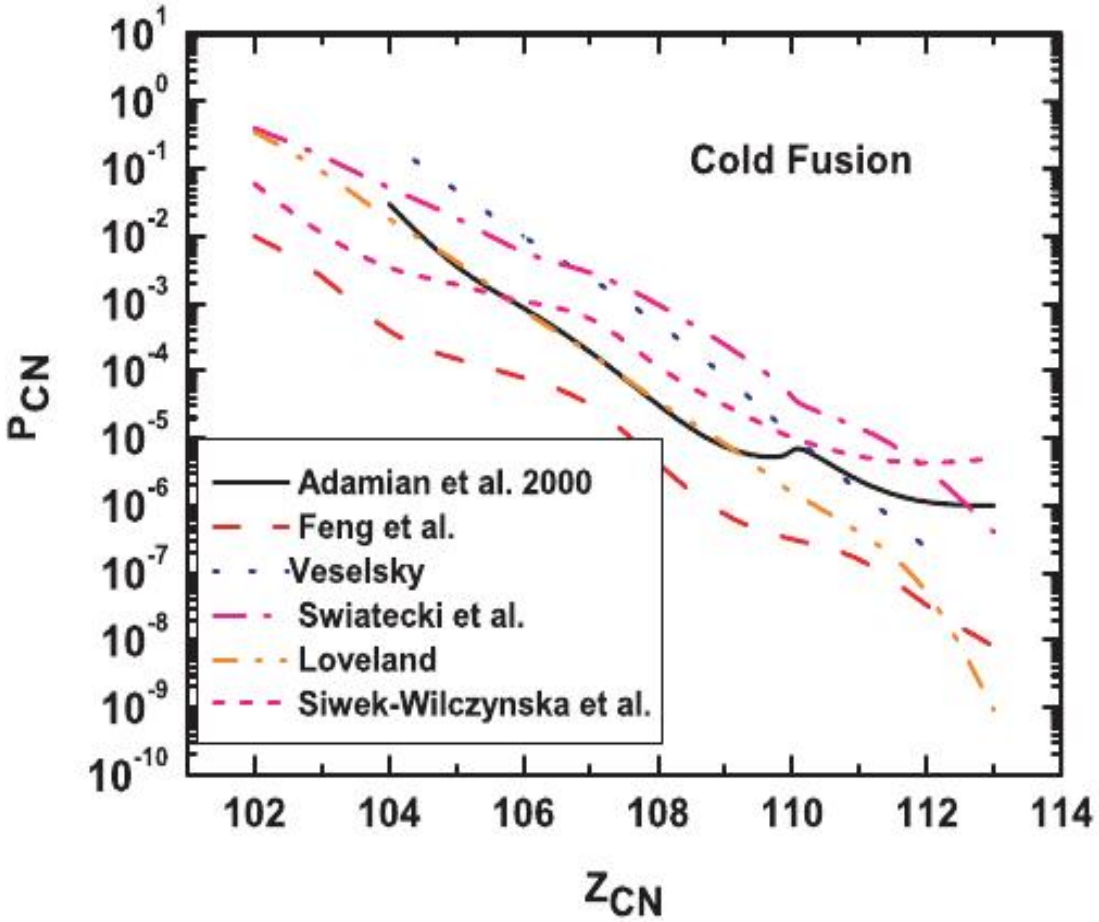
A. Marchix, PhD thesis (2007); H. LÜ *et al.*, in preparation for submission to CPC.

Fusion hindrance ... ?! An example

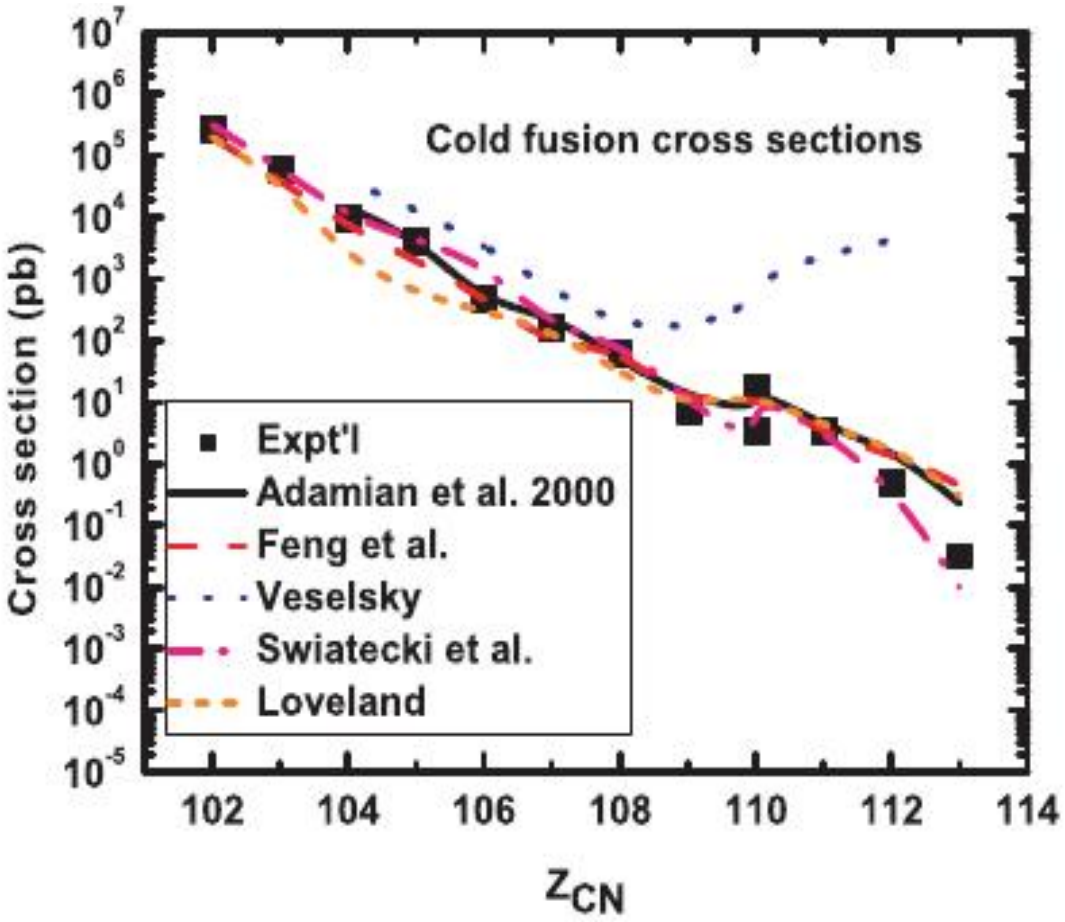


Large discrepancies among fusion models ...



Naik, Loveland et al, Phys. Rev. C 76, 054604.

But, predictions highly consistent with measurements!



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What can we learn from this delicate situation?

Questions:

- All fusion-evaporation models reproduce well the experimental data, however large discrepancies among formation probabilities, why?
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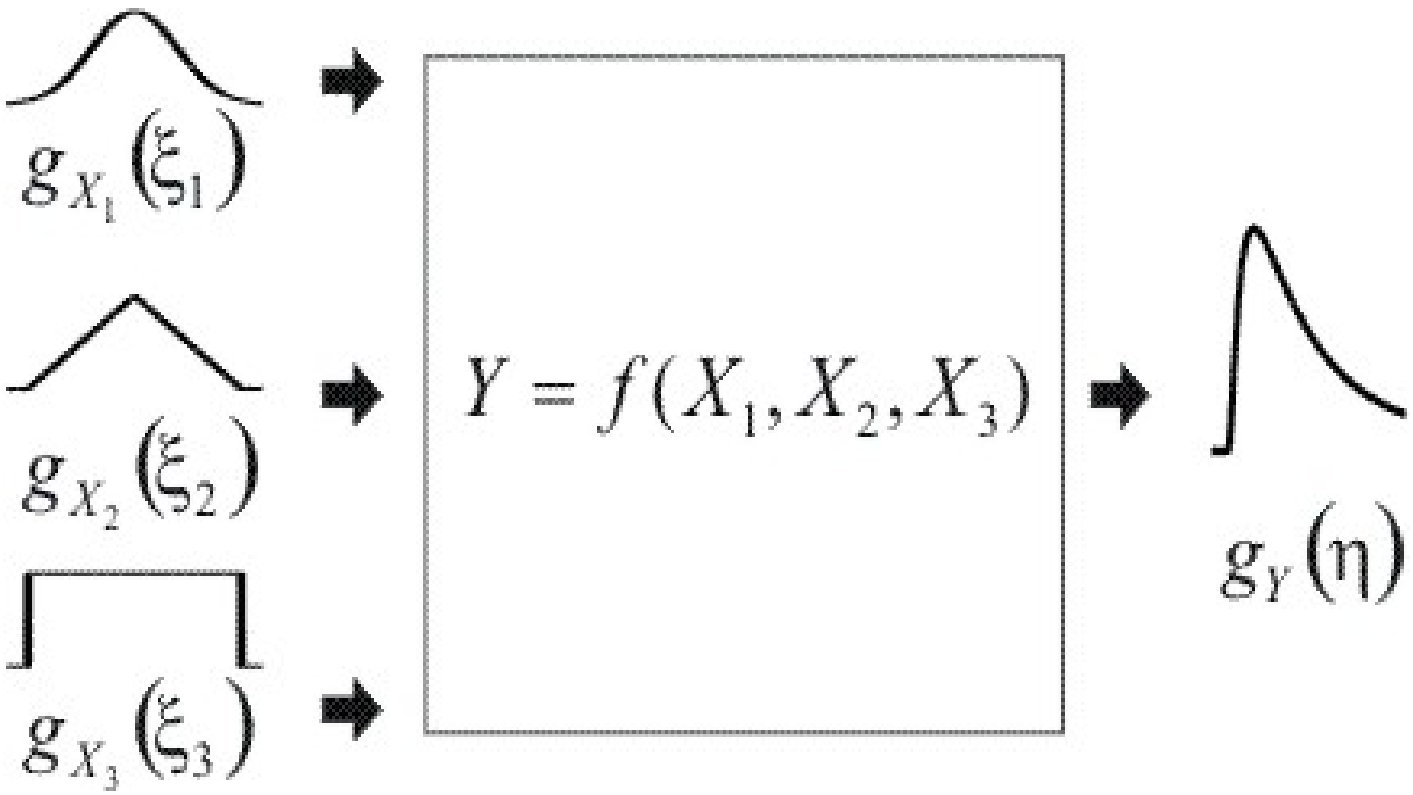
Solutions:

- Estimating the survival probability W_{sur} by code;
- Uncertainty analysis of P_{form} via $\dfrac{\sigma_{\mathrm{res}}^{1\mathrm{n}}(\mathrm{experimental})}{\sigma_{\mathrm{cap}} W_{\mathrm{sur}}(\mathrm{theoretical})}$.

KEWPIE2 + Uncertainty propagation (MCM, proposed in GUM-S1)

Propagation of Uncertainty (MCM)



The MCM determines numerically a probability density function (PDF) which encodes the knowledge about the quantity of interest. An estimate and its associated uncertainty are then determined as the expectation and standard deviation of this PDF.

In the present work

Uncertainty sources: Parameters and Models

Parameters:

- Experimental data (Gaussian distribution);
- Reduced friction coefficient $\beta \simeq 1.0 - 5.0\ zs^{-1}$ (flat distribution);
- Damping-shell factor $E_d \simeq 11.0 - 19.0$ MeV (flat distribution).

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Models:

- Kramers factor for fission-decay width;
- Collective enhancement factor for state density;
- Fission barriers taken from different models differ by $1 \sim 2$ MeV ($B_f \simeq B_{\mathrm{LDM}} - \Delta E_{\mathrm{sh}}$ or multidimensional calculation).

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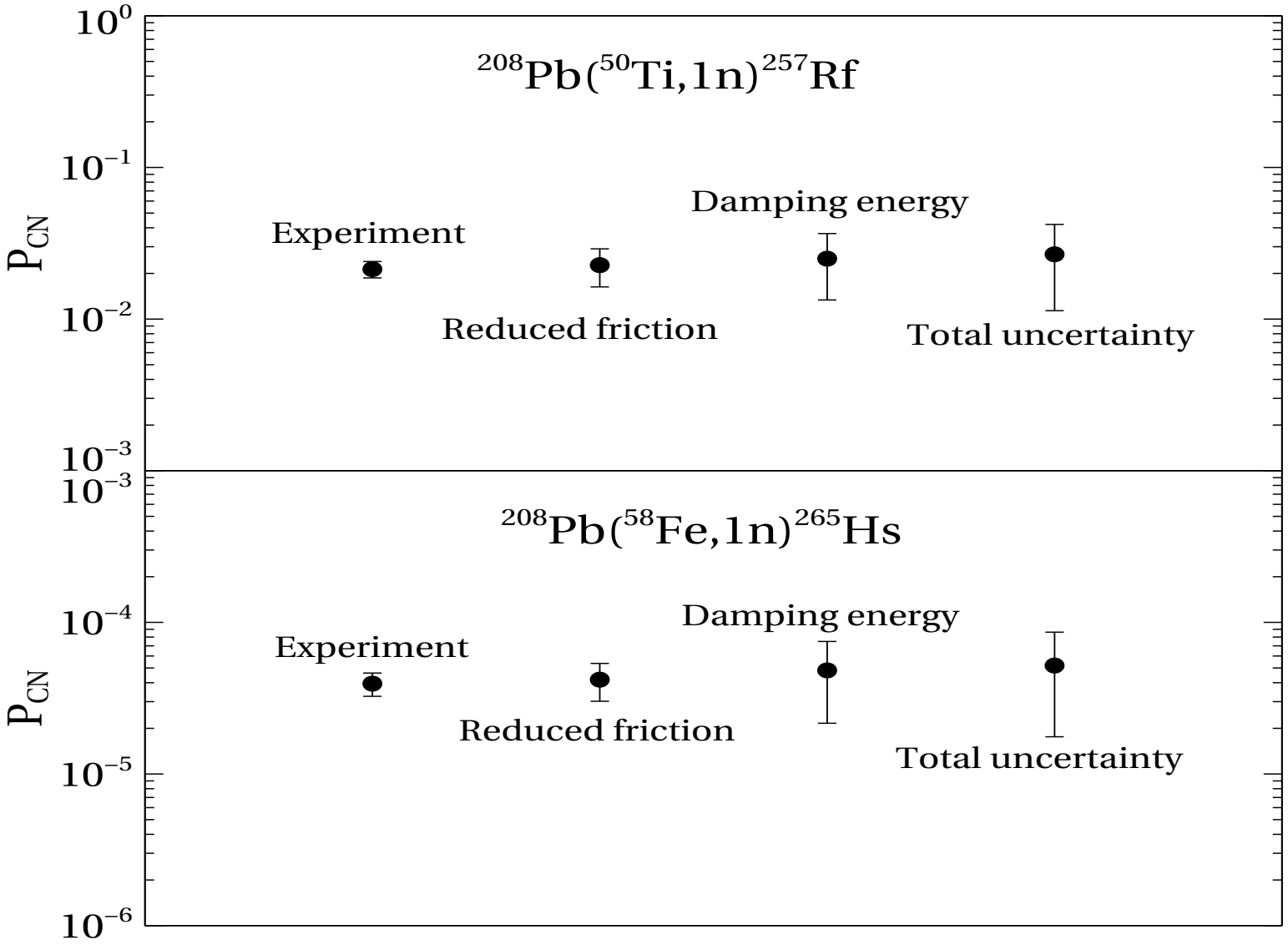
What will we get?

Parameters


^{208}Pb(^{50}Ti,1n)^{257}Rf
P_{CN}
10^{0}
10^{-1}
10^{-2}
10^{-3}
Experiment
Reduced friction
Damping energy
Total uncertainty
^{208}Pb(^{58}Fe,1n)^{265}Hs
P_{CN}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
Experiment
Reduced friction
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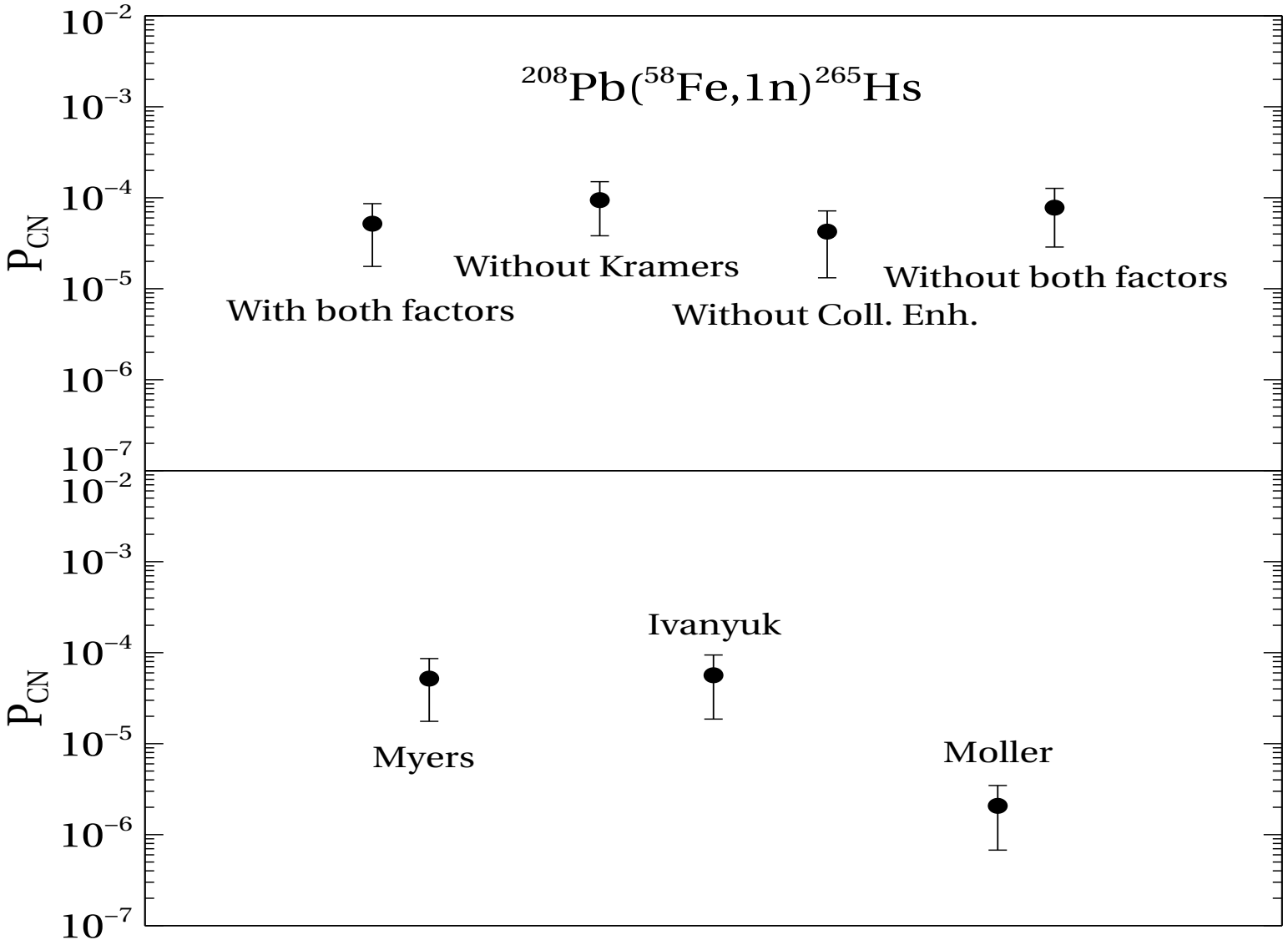
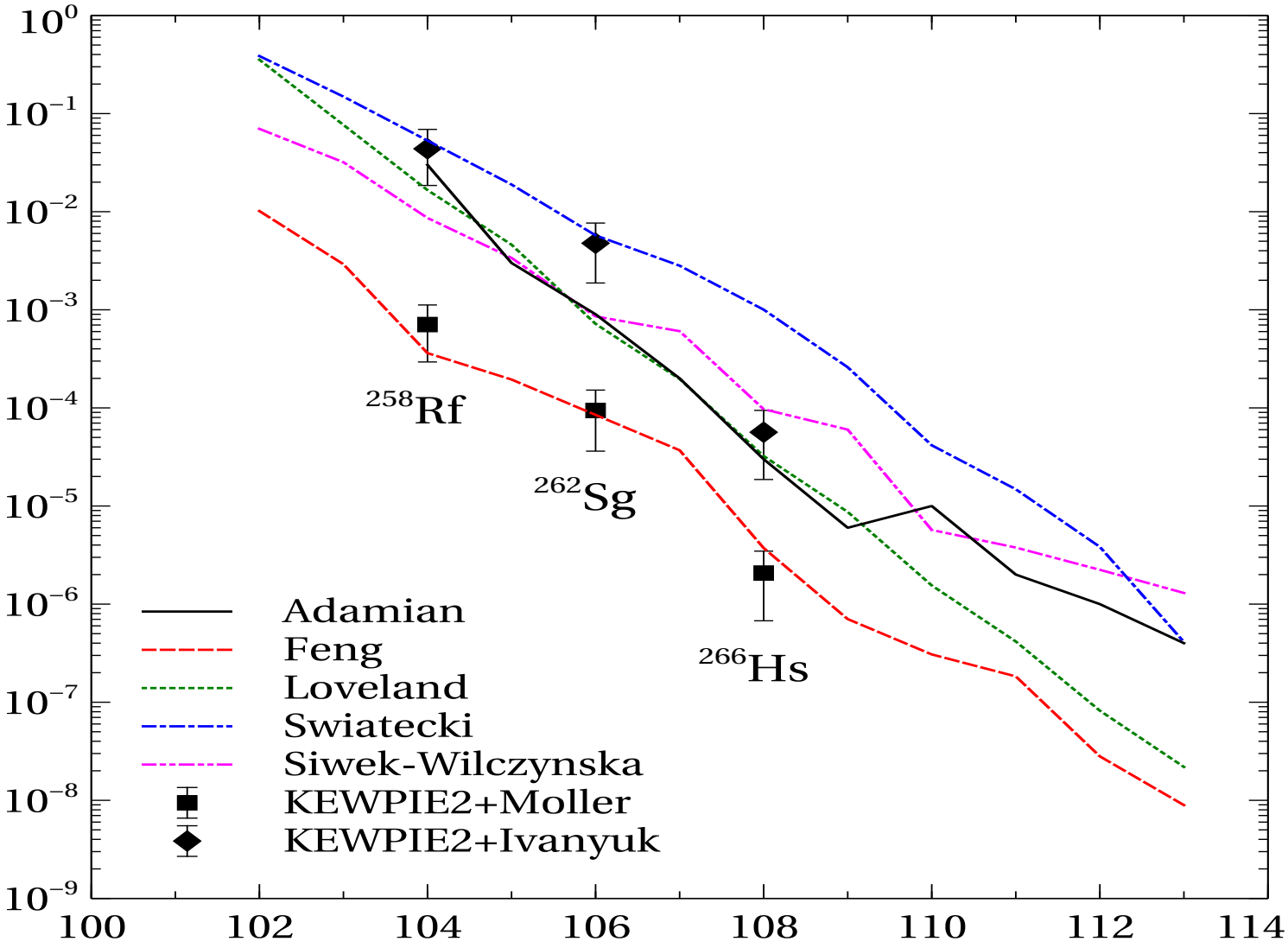
Models


^{208}Pb(^{58}Fe,1n)^{265}Hs
P_{CN}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
With both factors
Without Kramers
Without Coll. Enh.
Without both factors
Myers
Ivanyuk
Moller

Summary


10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
100
102
104
106
108
110
112
114
^{258}Rf
^{262}Sg
^{266}Hs
Adamian
Feng
Loveland
Swiatecki
Siwek-Wilczynska
KEWPIE2+Moller
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Conclusion

The uncertainty contribution from the decay step has the same order of magnitude as the fusion step ... How to access different fusion models?

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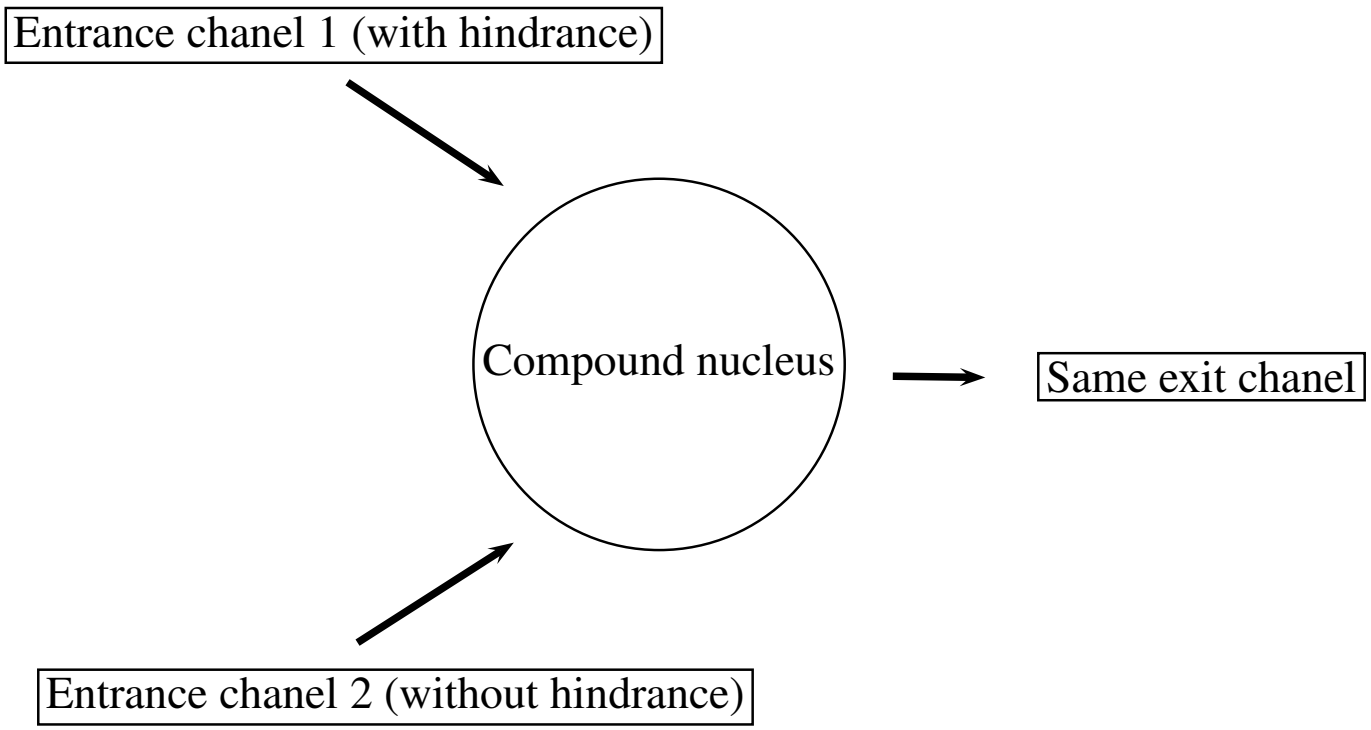

Hopefully, one can constrain P_{form} directly from experimental measurements.

How to infer B_f from experimental data?

Bayes rule:

$$P(\text{Parameters} \mid \text{Data}) = \frac{P(\text{Data} \mid \text{Parameters}) \cdot P(\text{Parameters})}{P(\text{Data})}$$

For light systems without fusion hindrance, the ER cross-section is given by

$$\sigma_{1\text{n}}^{\text{thr}} = \sigma_{\text{cap}} \frac{\Gamma_n}{\Gamma_n + \Gamma_f},$$

The posterior distribution $P(\text{Parameters} \mid \text{Data})$ is thus proportional to

$$\exp\left[-\frac{1}{2}(\mathbf{y}_{\text{exp}} - \mathbf{y}_{\text{thr}})^{\text{T}} \mathbf{X}^{-1} (\mathbf{y}_{\text{exp}} - \mathbf{y}_{\text{thr}})\right] \times P(\text{Parameters}),$$

where $\mathbf{y} = \{\sigma_{1\text{n}}\}$ and $\mathbf{X}$ the variance matrix. Prior distribution $P(\text{Parameters})$ is non-informative. $P(\text{Data})$ evaluated by Gauss quadrature or MC integration.

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From the posterior distribution, one can determine $P(B_f)$ by marginalization and thus the conditional expectation value and its uncertainty.

Thanks for your attention!