Formation of Super-Heavy Elements
— Uncertainties in theoretical modeling

Hongliang LÜ

GANIL, CEA/DSM-CNRS/IN2P3, Bd Henri Becquerel, 14076 Caen, France
Normandie Université, France

Supervisor and Co-supervisor: Abdelouahad CHBIHI and David BOILLEY.

December 16, 2014
Where is the "Island of Stability"? How to get there?
Where is the “Island of Stability”? How to get there?
Theoretical viewpoint

Schematic of fusion-evaporation reaction

Fusion-evaporation reaction:

Nuclear fission is the dominant decay channel of SHE formed in fusion-evaporation reactions. Typically, one has $B_{f} < S_{n}$. Re-separation (or quasi-fission) process only occurs in heavy binary systems. This phenomenon is usually called the "Fusion hindrance". Key open question for the synthesis of SHE!
Nuclear fission is the dominant decay channel of SHE formed in fusion-evaporation reactions. Typically, one has $B_f < S_n$. 

Fusion-evaporation reaction:

- Projectile + Target → Di-nuclear system
- Re-separation before fusion
- Fusion-fission (break-up of the compound nucleus)
- Compound nucleus
- Fusion product
Fusion-evaporation reaction:

Nuclear fission is the dominant decay channel of SHE formed in fusion-evaporation reactions. Typically, one has $B_f < S_n$.

... Re-separation (or quasi-fission) process only occurs in heavy binary systems. This phenomenon is usually called the “Fusion hindrance”. Key open question for the synthesis of SHE!
Evaporation-residue (ER) cross-section of SHE

\[ \sigma_{\text{res}} = \sigma_{\text{cap}} \times P_{\text{form}} \times W_{\text{sur}} \]
Theoretical viewpoint

Evaporation-residue (ER) cross-section of SHE

\[ \sigma_{\text{res}} = \sigma_{\text{cap}} \times P_{\text{form}} \times W_{\text{sur}} \]

For light systems, \( P_{\text{form}} = 1 \);

For heavy systems \((Z_t \cdot Z_p \gtrsim 1600 - 1800)\), \( P_{\text{form}} < 1 \).
How to synthesize SHE?

Modelling of statistical decay of SHE

\( W_{\text{sur}} \)? KEWPIE2 code ... For guiding experiments

KEWPIE2

- Level density
- Gamma decay
- Particle evaporation
- Nuclear fission
- Nuclear mass table
  ...

Basic features:
Not a Monte-Carlo cascade code, due to low survival probabilities.
Based on the discretization of the energy spectrum. More efficient!

A. Marchix, PhD thesis (2007); H. LÜ et al., in preparation for submission to CPC.
How to synthesize SHE?

Modeling of statistical decay of SHE

$W_{\text{sur}}$? KEWPIE2 code ... For guiding experiments

KEWPIE2

Basic features:

- Not a Monte-Carlo cascade code, due to low survival probabilities.
- Based on the discretization of the energy spectrum. More efficient!

- Level density
- Gamma decay
- Particle evaporation
- Nuclear fission
- Nuclear mass table

A. Marchix, PhD thesis (2007); H. LÜ et al., in preparation for submission to CPC.
How to synthesize SHE?  
Modeling of statistical decay of SHE

$W_{\text{sur}}$? KEWPIE2 code ... For guiding experiments

KEWPIE2

- Level density
- Gamma decay
- Particle evaporation
- Nuclear fission
- Nuclear mass table
- ...

Basic features:
- Not a Monte-Carlo cascade code, due to low survival probabilities.
- Based on the discretization of the energy spectrum. More efficient!

A. Marchix, PhD thesis (2007); H. LÜ et al., in preparation for submission to CPC.
Fusion hindrance ... ?! An example

\[ \sigma_{\text{fus}} = \sigma_{\text{cap}} \times P_{\text{form}}(J=0) \]

Exp. data from INDRA

Fusion without hindrance

Fusion with hindrance
Large discrepancies among fusion models ...

But, predictions highly consistent with measurements!

Current status of research on synthesis of SHE

What can we learn from this delicate situation?

Questions:
- All fusion-evaporation models reproduce well the experimental data, however large discrepancies among formation probabilities, why?
- Can we constrain formation probabilities by examining the decay step (better understood)?
What can we learn from this delicate situation?

Questions:
- All fusion-evaporation models reproduce well the experimental data, however large discrepancies among formation probabilities, why?
- Can we constrain formation probabilities by examining the decay step (better understood)?

Solutions:
- Estimating the survival probability $W_{\text{sur}}$ by code;
- Uncertainty analysis of $P_{\text{form}}$ via $\frac{\sigma_{\text{res}}^{\text{ln}}(\text{experimental})}{\sigma_{\text{cap}} W_{\text{sur}}(\text{theoretical})}$.

KEWPIE2 + Uncertainty propagation (MCM, proposed in GUM-S1)
The MCM determines numerically a probability density function (PDF) which encodes the knowledge about the quantity of interest. An estimate and its associated uncertainty are then determined as the expectation and standard deviation of this PDF.
In the present work

Uncertainty sources: Parameters and Models

<table>
<thead>
<tr>
<th>Parameters:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental data (Gaussian distribution);</td>
</tr>
<tr>
<td>Reduced friction coefficient $\beta \simeq 1.0 - 5.0 \text{ zs}^{-1}$ (flat distribution);</td>
</tr>
<tr>
<td>Damping-shell factor $E_d \simeq 11.0 - 19.0 \text{ MeV}$ (flat distribution).</td>
</tr>
</tbody>
</table>
In the present work

Uncertainty sources: Parameters and Models

**Parameters:**
- Experimental data (Gaussian distribution);
- Reduced friction coefficient $\beta \simeq 1.0 - 5.0 \, zs^{-1}$ (flat distribution);
- Damping-shell factor $E_d \simeq 11.0 - 19.0 \, \text{MeV}$ (flat distribution).

**Models:**
- Kramers factor for fission-decay width;
- Collective enhancement factor for state density;
- Fission barriers taken from different models differ by $1 \sim 2 \, \text{MeV}$ ($B_f \simeq B_{\text{LDM}} - \Delta E_{\text{sh}}$ or multidimensional calculation).
In the present work

Uncertainty sources: Parameters and Models

Parameters:
- Experimental data (Gaussian distribution);
- Reduced friction coefficient $\beta \simeq 1.0 - 5.0 \text{zs}^{-1}$ (flat distribution);
- Damping-shell factor $E_d \simeq 11.0 - 19.0 \text{MeV}$ (flat distribution).

Models:
- Kramers factor for fission-decay width;
- Collective enhancement factor for state density;
- Fission barriers taken from different models differ by $1 \sim 2 \text{MeV}$ ($B_f \simeq B_{\text{LDM}} - \Delta E_{\text{sh}}$ or multidimensional calculation).

What will we get?
Parameters

\[ {^{208}}\text{Pb}^{(58}\text{Fe}, 1\text{n})^{265}\text{Hs} \]

\[ {^{208}}\text{Pb}^{(50}\text{Ti}, 1\text{n})^{257}\text{Rf} \]

- **Experiment**
- **Damping energy**
- **Reduced friction**
- **Total uncertainty**

**Uncertainty analysis**

**Results**

**Parameters**
Results

Models

$^{208}\text{Pb}(^{58}\text{Fe},1\text{n})^{265}\text{Hs}$

- Without both factors
- Without Coll. Enh.
- Without Kramers
- With both factors

$P_{CN}$ vs. $P_{\text{CN}}$

- Moller
- Ivanyuk
- Myers
- Moller

Hongliang LÜ (GANIL, Caen Univ.)

Formation of Super-Heavy Elements

December 16, 2014

13 / 20
Summary

The graph shows the formation of super-heavy elements with different isotopes labeled.

- **258-Rf**
- **262-Sg**
- **266-Hs**

The lines represent different authors:
- Adamian
- Feng
- Loveland
- Swiatecki
- Siwek-Wilczynska
- KEWPIE2+Moller
- KEWPIE2+Ivanyuk

Values are shown on a logarithmic scale, ranging from $10^{-9}$ to $10^0$.
The uncertainty contribution from the decay step has the same order of magnitude as the fusion step ... How to access different fusion models?
The uncertainty contribution from the decay step has the same order of magnitude as the fusion step... How to access different fusion models?

Hopefully, one can constrain $P_{\text{form}}$ directly from experimental measurements.
Bayesian inference

How to infer $B_f$ from experimental data?

Bayes rule:

$$P(\text{Parameters} \mid \text{Data}) = \frac{P(\text{Data} \mid \text{Parameters}) \cdot P(\text{Parameters})}{P(\text{Data})}$$

For light systems without fusion hindrance, the ER cross-section is given by

$$\sigma_{1n}^{\text{thr}} = \sigma_{\text{cap}} \frac{\Gamma_n}{\Gamma_n + \Gamma_f},$$

The posterior distribution $P(\text{Parameters} \mid \text{Data})$ is thus proportional to

$$\exp \left[ -\frac{1}{2} (y_{\exp} - y_{\text{thr}})^T X^{-1} (y_{\exp} - y_{\text{thr}}) \right] \times P(\text{Parameters}),$$

where $y = \{\sigma_{1n}\}$ and $X$ the variance matrix. Prior distribution $P(\text{Parameters})$ is non-informative. $P(\text{Data})$ evaluated by Gauss quadrature or MC integration.
Bayes rule:

\[ P(\text{Parameters} \mid \text{Data}) = \frac{P(\text{Data} \mid \text{Parameters}) \cdot P(\text{Parameters})}{P(\text{Data})} \]

For light systems without fusion hindrance, the ER cross-section is given by

\[ \sigma_{1n}^{\text{thr}} = \sigma_{\text{cap}} \frac{\Gamma_n}{\Gamma_n + \Gamma_f}, \]

The posterior distribution \( P(\text{Parameters} \mid \text{Data}) \) is thus proportional to

\[ \exp\left[ -\frac{1}{2}(y_{\text{exp}} - y_{\text{thr}})^T X^{-1} (y_{\text{exp}} - y_{\text{thr}}) \right] \times P(\text{Parameters}), \]

where \( y = \{\sigma_{1n}\} \) and \( X \) the variance matrix. Prior distribution \( P(\text{Parameters}) \) is non-informative. \( P(\text{Data}) \) evaluated by Gauss quadrature or MC integration.

From the posterior distribution, one can determine \( P(B_f) \) by marginalization and thus the conditional expectation value and its uncertainty.
Thanks for your attention!