Assemblée Générale des Théoriciens

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Proto-Neutron Star Models with Finite Temperature Equation of State

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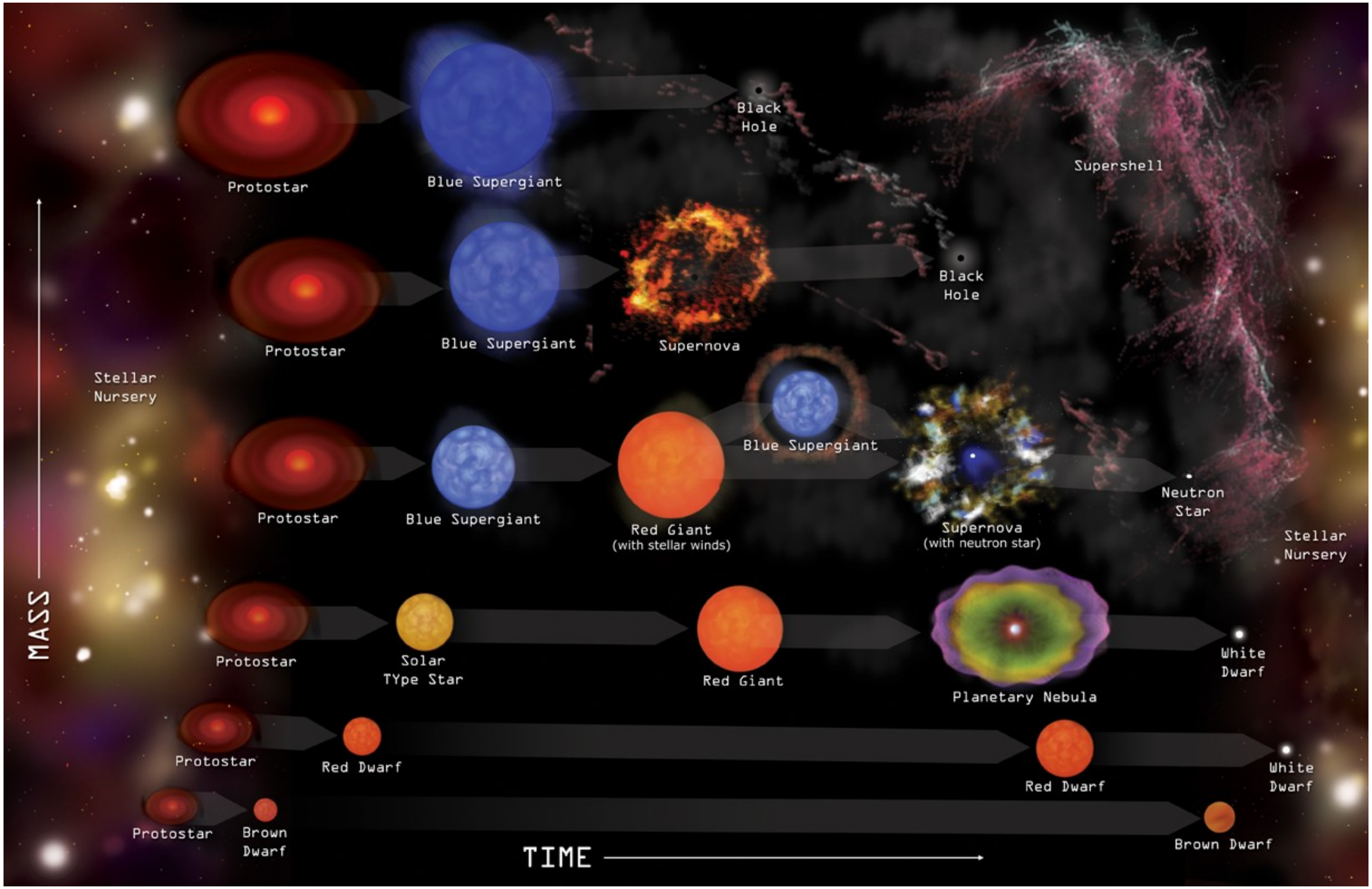
How do Massive Stars Die?

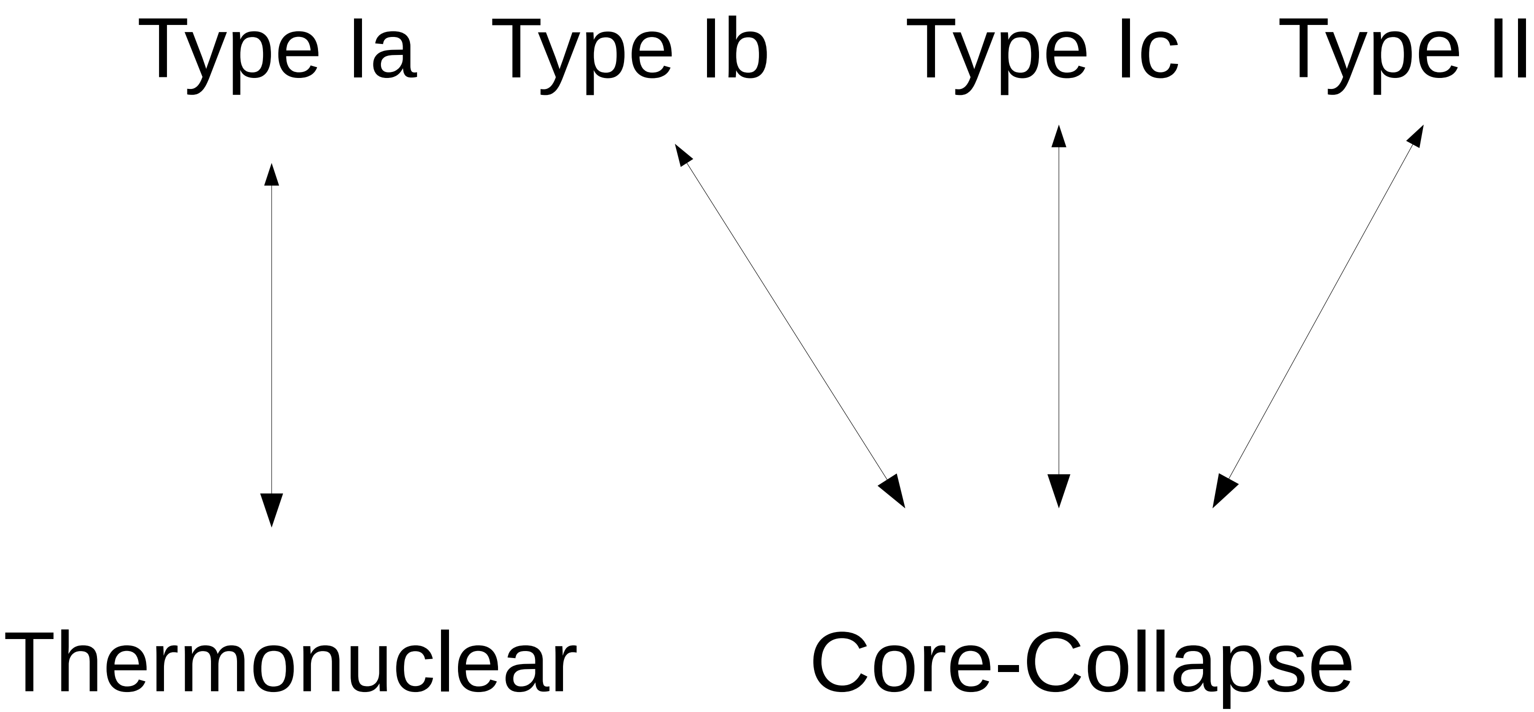
- The final fate of star evolution



Chandra

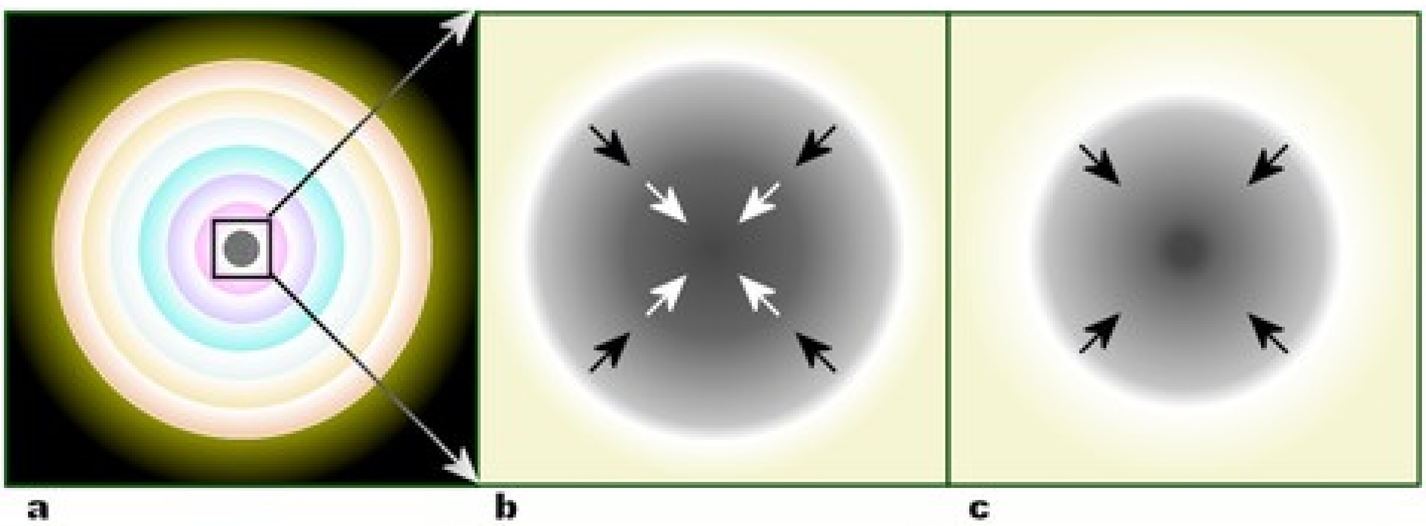
How do Massive Stars Die?

- Which supernovae?



Core-Collapse

- The gravitational collapse of the iron core



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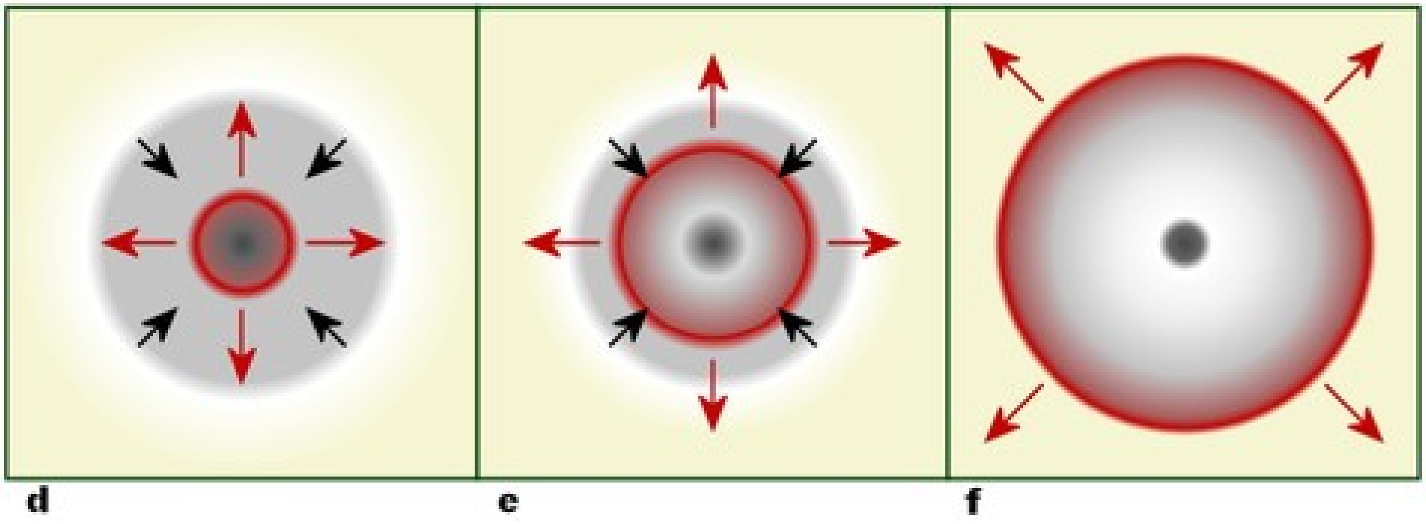
Core-Collapse

- The consequences of the core-collapse



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Core Collapse

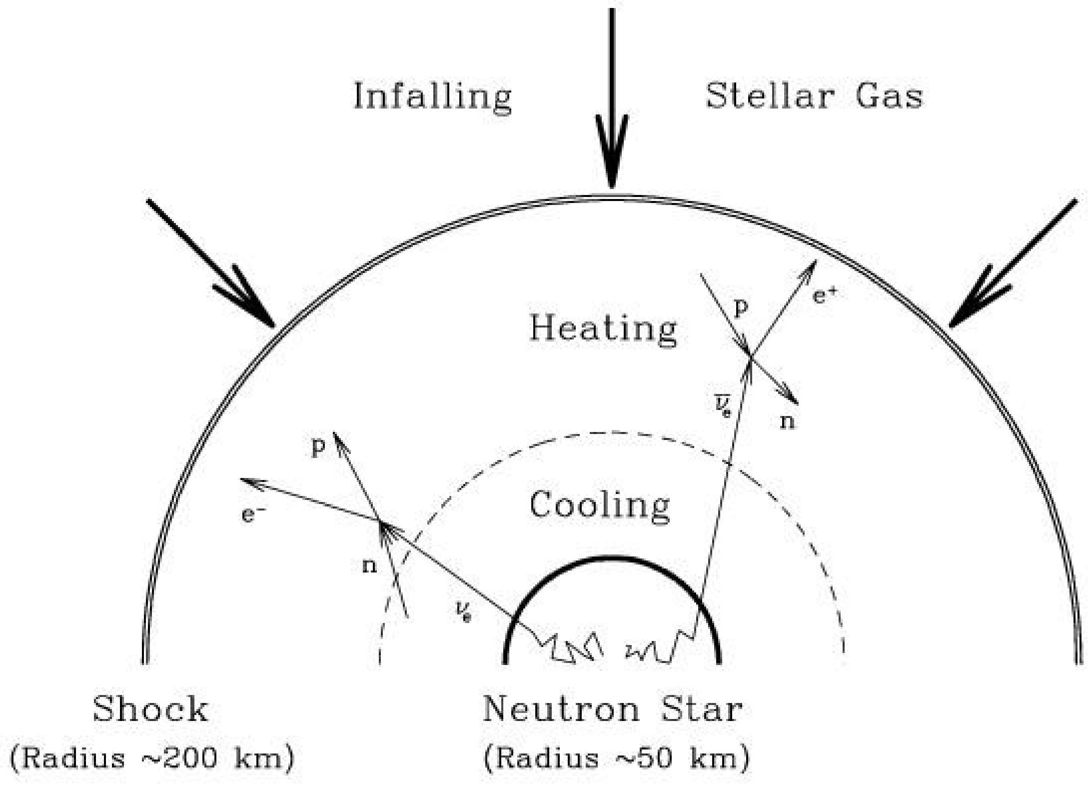
- The neutrino driven mechanism

Explosion mechanisms

- The neutrino heating
- Hydrodynamical instabilities (Sasi, convection)
- Magnetorotational instabilities

Depends on ingredients from

- General relativity
- Microphysics input

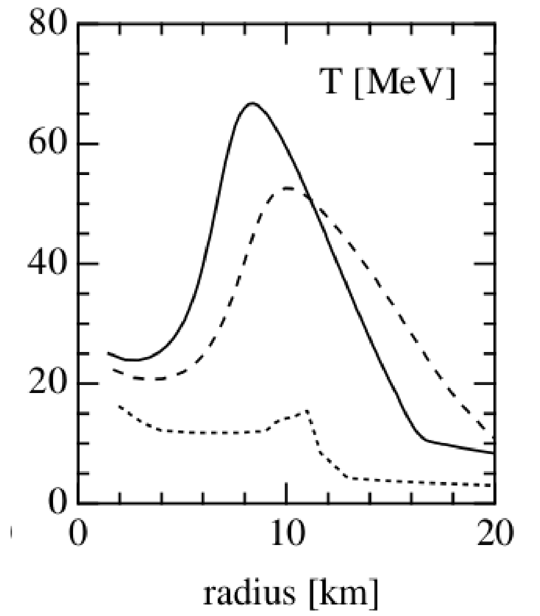


H. T. Janka

Finite Temperature in Compact Stars

- Core-Collapse Supernovae

  - T ~ 50 - 100 MeV



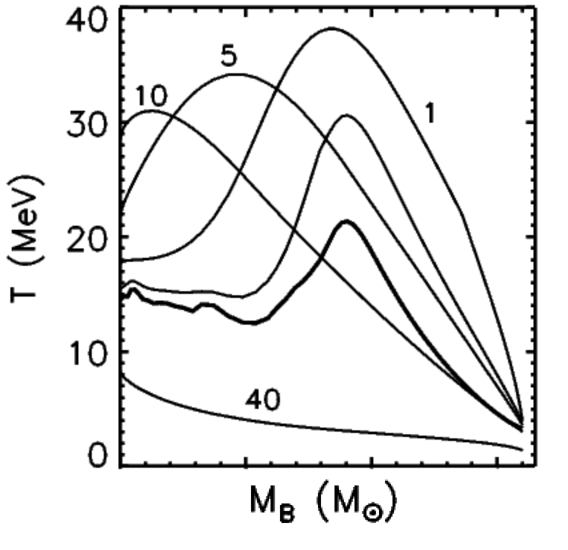
Fischer (2009)

- Neutron Star Mergers

  - T ~ 80 MeV

- Proto-Neutron Stars

  - T ~ 50 MeV



Ferrari et al (2003)

Finite Temperature in Compact Stars

- Core-Collapse Supernovae

  - Temperature effects included since many years in dynamical simulations

- Neutron Star Mergers

  - Kaplan et al (2014), Bauswein et al (2012), Abdikamalov et al (2013), Sekiguchi et al (2011) …

- Proto-Neutron Stars

  - Martinon et al (2014), Villain et al (2004), Pons et al (1999), Goussard et al (1998, 1997) ...

Finite Temperature in Compact Stars

- Finding a general solution for the equilibrium equations of stationary axisymetric spacetimes with finite temperature is difficult. Typical strategies are:

- Effective barotropic EoS

Goussard et al (97,98), ...

$$p = p(n_b, T(n_b), Y_e(n_b, T(n_b)))$$

- Using perturbative approximation methods for the metric

Martinon et al (2014)

Our model for hot stars

- General relativistic, stationary axisymmetric solutions of rotating stars

- Perfect fluid

Energy-momentum tensor → $T^{\mu\nu} = (p+\varepsilon)u^\mu u^\nu + p\, g^{\mu\nu}$

- Temperature (or entropy) dependent EoS's

$$p := p(n_b, s_b)$$
$$\varepsilon := \varepsilon(n_b, s_b)$$

Solving the star structure

- The TOV equations

Tolman, (1934)

Oppenheimer and Volkoff, (1939)

$$\partial_r m = 4\pi r^2 \varepsilon$$

← enclosed mass

$$\partial_r p = -(\varepsilon + p)\frac{m + 4\pi r^3 p}{r(r-2m)}$$

← equilibrium equation

$$\partial_r \nu = -\frac{1}{\varepsilon + p}\partial_r p$$

← metric potential

With an EoS, these equations will yield the line element for the star interior

$$ds^2 = -e^{2\nu}dt^2 + \left(1 - \frac{2m(r)}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

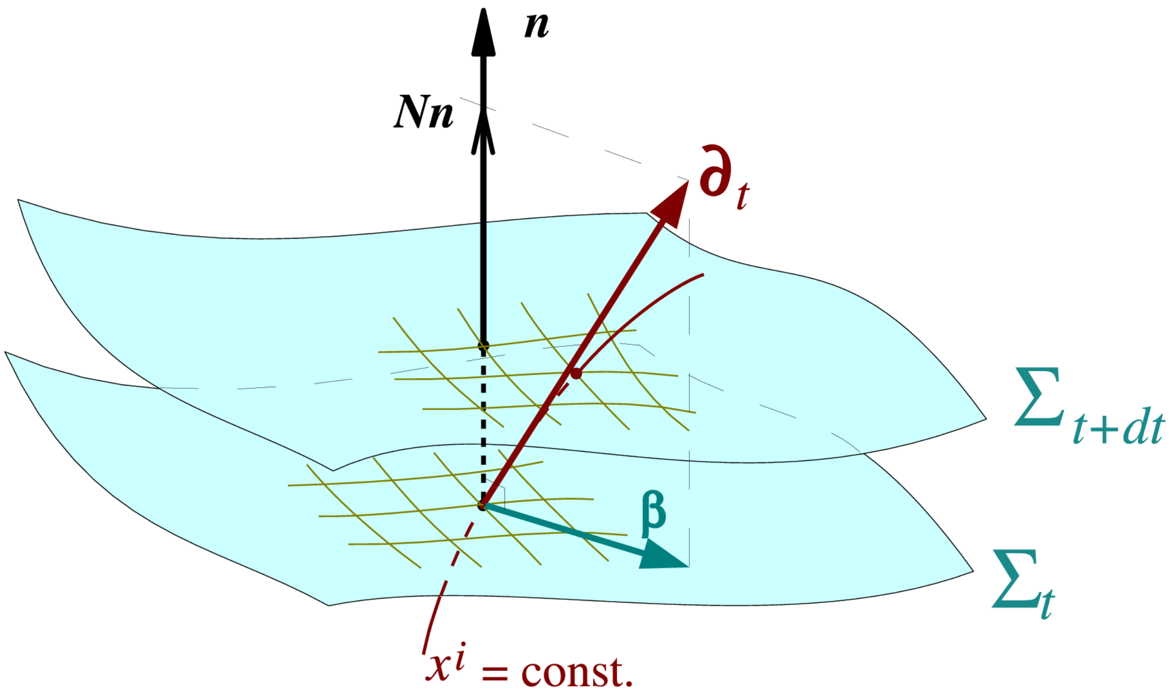
and Schwarzschild outside the star. These are **stationary, spherically symmetric** stars

Solving the star structure

- The space-time solution

We assume two symmetries: stationarity and axisymmetry;

We use the conformal metric (Bonazzola et al, 2004), imposing Dirac gauge and maximal slicing condition.



E.Gourgoulhon (2012)

$$ds^2 = -N^2 dt^2 + \gamma_{ij}\left(dx^i + \beta^j dt\right)\left(dx^j + \beta^i dt\right)$$

Star equilibrium

- Equilibrium equations (for rigid rotation)

$$\nabla_\mu T^{\mu\nu} = 0$$

Defining the pseudo-log enthalpy field as

$$H = \ln\left(\frac{\varepsilon + p}{m_b n_b}\right),$$

With the help of the 1st law of thermodynamics, the equilibrium equations are

$$\partial_i \left(H + \nu - \ln\Gamma\right) = T e^{-H} \partial_i s_b, \qquad i = r, \theta$$

Goussard et al, 97

Star equilibrium

- Equilibrium equations (for rigid rotation)

$$\partial_i \left(H + \nu - \ln \Gamma \right) = T e^{-H} \partial_i s_b, \qquad i = r, \theta$$

No first integral in general! First integral for the barotropic EoS $H(n_b)$.

Star equilibrium

- Equilibrium equations (for rigid rotation)

$$\partial_i \left(H + \nu - \ln\Gamma\right) = T e^{-H} \partial_i s_b, \qquad i = r, \theta$$

Instead of solving an analytical first integral, we propose the following scheme

$$H = -\nu + \ln\Gamma + \int_0^{r_s} T e^{-H} \partial_r s_b \, dr,$$

$$\left(\partial_{\theta,\theta} + \frac{1}{\tan\theta}\partial_\theta\right) s_b = \partial^\theta \left(\frac{e^H}{T}\partial_\theta \left(H + \nu - \ln\Gamma\right)\right)$$

Where the monopolar part of the s_b has to be specified, whereas the higher multipoles are determined by the equilibrium solver

Results

- To test the code, we use the relativistic ideal gas EoS:

$$p(H, s_b) = k\, n_b(H, s_b)^\gamma e^{(\gamma-1)s_b}$$

$$\varepsilon(H, s_b) = \frac{k}{\gamma - 1} n_b(H, s_b)^\gamma e^{(\gamma-1)s_b} + m_b n_b(H, s_b)$$

$$T(H) = m_b \frac{\gamma - 1}{\gamma\, k_b} \left(e^H - 1\right)$$

$$n_b(H, s_b) = \left(m_b \frac{\gamma - 1}{\gamma\, k} \left(e^H - 1\right)\right)^{\frac{1}{\gamma-1}} e^{-s_b}$$

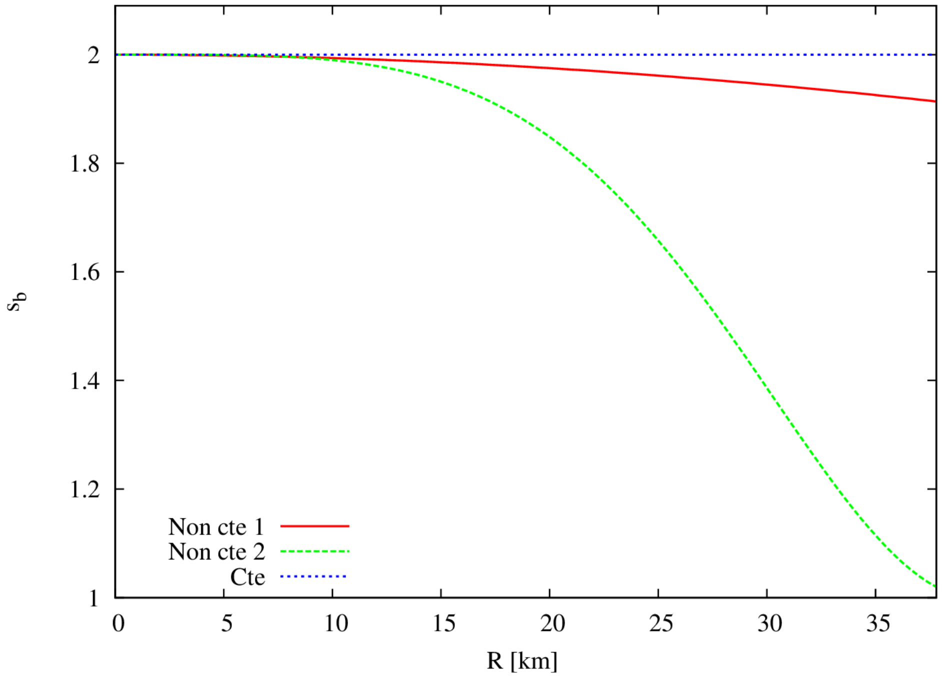
with $\gamma = 2$ and $k = 0.04\ \rho_{nuc}\ c^2 / n_{nuc}$

Results

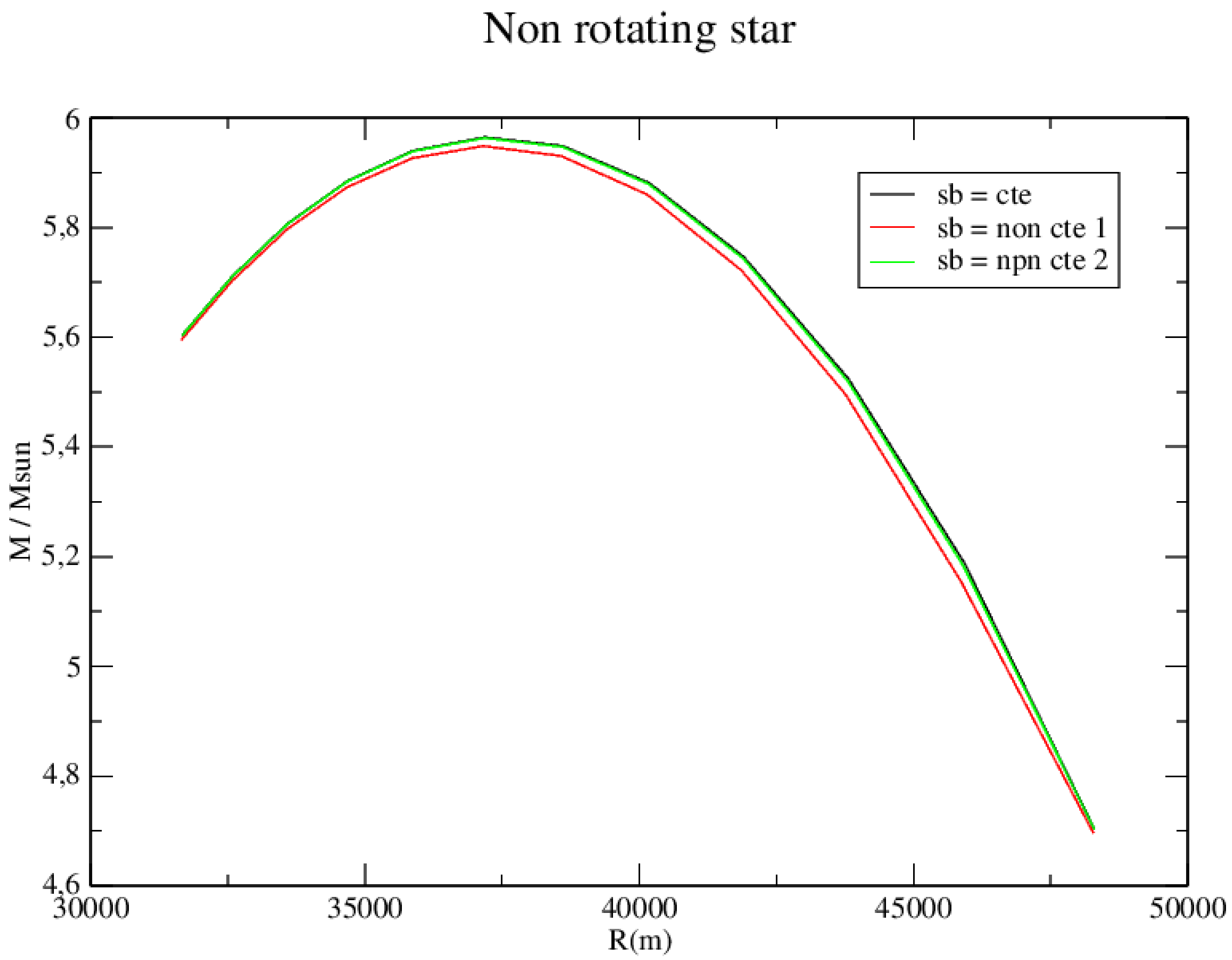
- We will consider two (ad hoc) non-constant entropy per baryon profiles

non cte 1 → $s_b = 1 + e^{-\frac{r^2}{10^{4.2}}}$

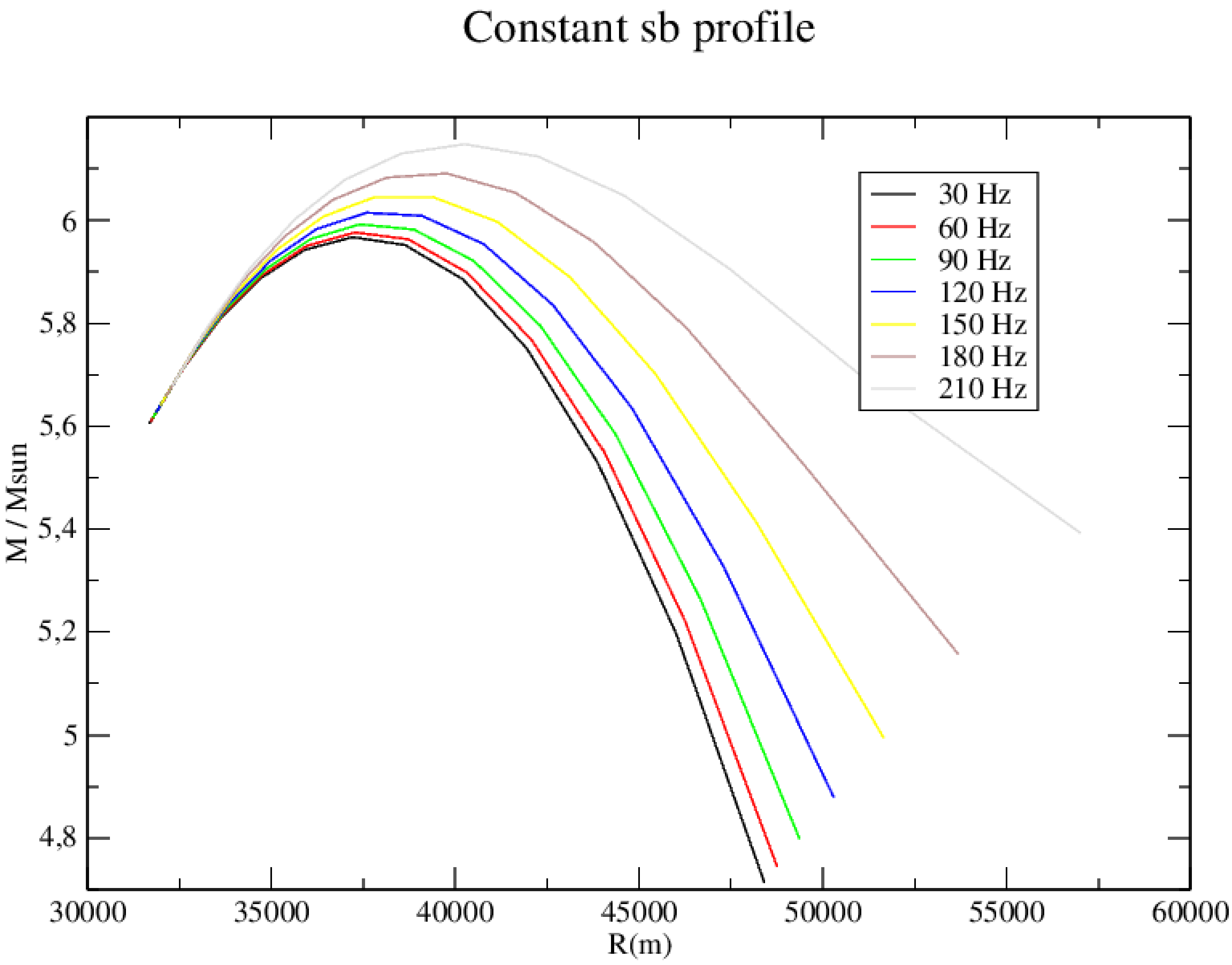
non cte 2 → $s_b = 1 + \cos\left(\frac{r^2}{10^3}\right)^2$



Results


Non rotating star
6
5,8
5,6
5,4
5,2
5
4,8
4,6
M / Msun
30000
35000
40000
45000
50000
R(m)
sb = cte
sb = non cte 1
sb = npn cte 2

Results


Constant sb profile
M / Msun
R(m)
30 Hz
60 Hz
90 Hz
120 Hz
150 Hz
180 Hz
210 Hz
6
5,8
5,6
5,4
5,2
5
4,8
30000
35000
40000
45000
50000
55000
60000

Conclusions and future perspectives

- We propose a numerical scheme to consistently make use of not necessarily barotropic EoS capable of modeling appropriately the finite temperature effects on a stationary axisymmetric star, using general entropy profiles
- Having in mind the use of realistic EoS's, we intend to extend the code for EoS's with electron fraction as a third parameter
- We plan to use this for developing quasi-stationary models of proto-neutron stars

Thank You!

Spacetime Solution

- Stationary axisymmetric spacetimes

Two killing vector fields:

$$\partial_t,\ \partial_\phi$$

We use a conformal metric, defined as:

$$\tilde{\gamma}_{ij} := \Psi^{-4}\gamma_{ij}$$

- Dirac Gauge

Bonazzola et al, (2004)

Lin and Novak, (2006)

$$\mathcal{D}_j h_{ij} = 0,$$

$$h_{ij} := \tilde{\gamma}^{ij} - f^{ij}$$

- Maximal slicing condition

$$K = 0$$

Spacetime Solution

- Metric calculations in Dirac gauge

Bonazzola et al, (2004)
Lin and Novak, (2006)

$$\Delta N = \sigma 1$$
$$\Delta \beta_i = \sigma 2$$
$$\Delta \Psi = \sigma 3$$
$$\Delta h_{ij} = \sigma 4$$