Octet baryon masses in covariant baryon chiral perturbation theory

Xiu-Lei Ren

School of Physics and Nuclear Energy Engineering, Beihang University

Institut de Physique Nucléaire, IN2P3-CNRS and Université Paris-Sud

Collaborators:

Li-Sheng Geng (BUAA), Jie Meng (PKU & BUAA),

J. Martin-Camalich (U.C. San Diego), and H. Toki (RCNP)

*PRD**84**(2011)074024, JHEP**12**(2012)073, PRD**87**(2013)074001, Eur.Phys.J. C**74**(2014)2754, arXiv:1404.4799*

OUTLINE

- Introduction
- Theoretical Framework
- Numerical Details
- Results and Discussion
- Summary

Origin of masses

☐ Current quark masses --- Explained

- Standard Model $\rightarrow$ Higgs Mechanism
- LHC @ CERN $\rightarrow$ Higgs particle

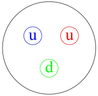
ATLAS Collaboration, PLB716(2012)1

CMS Collaboration, PLB716(2012)30

Nobel Prize 2013

☐ Light hadron masses --- Complicated

QCD



Proton

$M_u \sim 3$ MeV

$M_d \sim 6$ MeV

$M_p = 938$ MeV

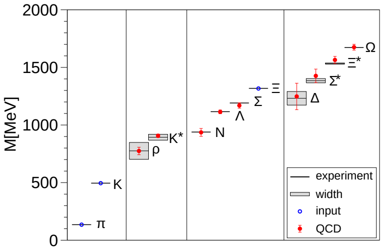
(Strong force)

$$M_p\ (938\text{MeV}) \gg m_u + m_u + m_d\ (12\text{MeV})$$

- Current quark masses (1-3%)
- Non-perturbative strong interaction (>95%)
  - Lattice QCD
  - Chiral Perturbation Theory
  - Other Models

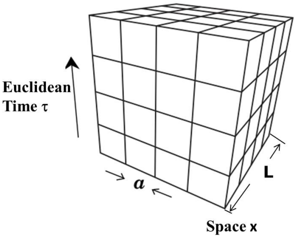
Octet baryon masses in LQCD

- $N_f = 2 + 1$ lattice calculation
  - BMW, *S. Dürr et al., Science 322 (2008) 1224*
  - PACS-CS, *S. Aoki et al., PRD 79 (2009) 034503*
  - LHPC, *A. Walker-Loud et al., PRD 79 (2009) 054502*
  - HSC, *H.-W. Lin et al., PRD 79 (2009) 034502*
  - UKQCD, *W. Bietenholz et al., PRD 84 (2011) 054509*
  - NPLQCD, *S. Beane et al., PRD 84 (2011) 014507*



☞Test the consistency --- **crucial**

- Lattice simulations:
  - **different fermion/gauge actions**
  - **different quark masses**
  - **different lattice volumes ($V = L^3$)**
  - **different lattice spacings (a)**
- In continuum:
  should lead to the same theory --- QCD



LQCD supplemented BChPT

- Cost of LQCD

$$\textbf{Cost} \propto \left(\frac{L}{a}\right)^4 \frac{1}{a} \frac{1}{m_{u/d}a}.$$

- Limitation of LQCD

Input of LQCD	Simulation	Physical World
Light quark masses $m_{u/d}$	~ 10 MeV	$3-5$ MeV
Lattice box size L	$2-5$ fm	Infinite space time
Lattice spacing a	$a \sim 0.1$fm	Continuum

In order to obtain the physical values

$$m_{u/d}^{\text{Lat.}} \xrightarrow{\textbf{Chiral extrapolation}} m_{u/d}^{\text{Phys.}}$$

$$L \xrightarrow{\textbf{Finite-volume corrections}} \infty$$

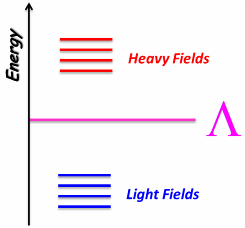
$$a \xrightarrow{\textbf{Continuum extrapolation}} 0$$

Baryon Chiral Perturbation Theory (BChPT) is a powerful tool to perform **the multi-extrapolation** for LQCD simulations.

Baryon Chiral Perturbation Theory

☐ Effective Field Theory of low-energy QCD

- Degrees of freedom
  - ✓ Pseudoscalar mesons, ✓ Baryons (Octet and Decuplet)
- Chiral symmetry $SU(3)_L \times SU(3)_R$
- Explicit and spontaneous symmetry breaking



☐ Solving the Power Counting Breaking problem

Non-Relativistic	Relativistic	
Heavy-Baryon ChPT *E.E. Jenkins et al., PLB(1991)*	Infrared BChPT *T. Becher et al., EPJC(1999)*	Extended-on-mass-shell (EOMS) *J. Gegelia et al., PRD(1999), T. Fuchs et al., PRD(2003)*
Baryon as static source Strict power-counting	$H = I + R$ $\int_0^1 \cdots = \int_0^\infty \cdots - \int_1^\infty \cdots$	PCB terms subtracted Redefinition of the LECs
breaks analyticity converges slowly	breaks analyticity converges slowly	satisfies analyticity converges relatively fast

Octet baryon masses in BChPT

Up to NNLO

- Heavy Baryon ChPT
  - ☞ **failed** to describe the lattice data *PACS-CS,PRD(2009), LHPC,PRD(2009)*
- EOMS-BChPT
  - ☞ **Improved description** of the PACS-CS and LHPC data *J. Martin-Camalich et al., PRD(2010)*
  - ☞ Finite-volume effects in LQCD simulations are very important *L.S. Geng et al., PRD(2011)*
- Finite-range regularization + HBChPT
  - ☞ nice description of the PACS-CS and LHPC data *R.D. Young et al., PRD(2010)*

Up to N^{3}LO --- **Few** calculations

Many low-energy constants (LECs) need to be fixed

- Partial summation BChPT
  - ☞ nice description of the BMW, PACS, LHPC and UKQCD data *A. Smeke et al., PRD(2012), M.F.M. Lutz et al., PRD(2013)*
- Infrared BChPT
  - ☞ nice description of UKQCD data *P.C. Bruns et al.,PRD(2013)*

In this work

Calculate the octet baryon masses in the EOMS BChPT up to N^{3}LO to systematically study the LQCD data

- ☑ Take into account finite volume corrections (FVCs) **self-consistently**
- ☑ Perform **a simultaneous fit** of all the $N_f = 2 + 1$ lattice results
  - ☞ Fix LECs and perform **chiral extrapolation**
  - ☞ Test the consistency of different LQCD data
- ☑ Perform the **continuum extrapolation** of LQCD up to $\mathcal{O}(a^2)$
  - ☞ Evaluate the finite lattice spacing discretization effects
- ☑ **Accurately** predict the sigma terms of octet baryon

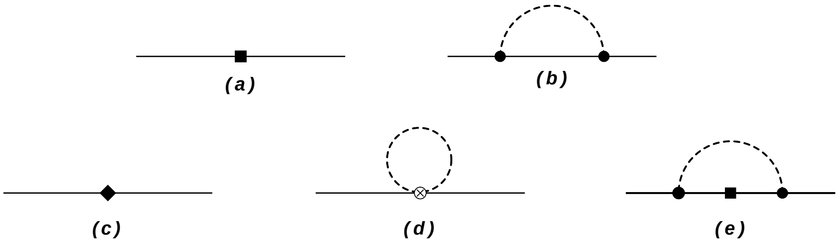
Theoretical Framework

Effective Lagrangians up to N^{3}LO

$$\mathcal{L}_{\rm eff} = \mathcal{L}_\phi^{(2)} + \mathcal{L}_\phi^{(4)} + \mathcal{L}_{\phi B}^{(1)} + \mathcal{L}_{\phi B}^{(2)} + \mathcal{L}_{\phi B}^{(3)} + \mathcal{L}_{\phi B}^{(4)}$$
$$= \frac{{F_\phi}^2}{4}\langle D_\mu U (D_\mu U)^\dagger\rangle + \frac{{F_\phi}^2}{4}\langle \chi U^\dagger + U\chi^\dagger\rangle + \sum_{i=4}^{8} L_i \hat{\mathcal{O}}_\phi^{(4)}$$
$$+ \langle \bar{B}(i\not{D} - M_0)B\rangle + \frac{D/F}{2}\langle \bar{B}\gamma^\mu\gamma_5[u_\mu, B]_\pm\rangle$$
$$+ b_0\langle\chi_+\rangle\langle B\bar{B}\rangle + b_{D/F}\langle\bar{B}[\chi_+, B]_\pm\rangle + \sum_{j=1}^{8} b_j\hat{\mathcal{O}}_{\phi B}^{(2)} + \sum_{k=1}^{7} d_k\hat{\mathcal{O}}_{\phi B}^{(4)}.$$

- The meson Lagrangians. *Gasser,NPB(1985)*
  - LECs from $\mathcal{L}_\phi^{(2)}$, $\mathcal{L}_\phi^{(4)}$: F_ϕ, L_i, $i \in (4,5,6,7,8)$
- The meson-baryon Lagrangians. *Borasoy,A.P.(1996), Oller,JHEP(2006)*
  - LECs from $\mathcal{L}_{\phi B}^{(1)}$: m_0, D, F
  - LECs from $\mathcal{L}_{\phi B}^{(2)}$: $b_0, b_D, b_F, b_j,\ j \in (1,\cdots,8)$
  - LECs from $\mathcal{L}_{\phi B}^{(4)}$: $d_k,\ k \in (1,\cdots,7)$

Feynman diagrams up to N^{3}LO



Fields: *Solid lines --- octet-baryons, Dashed lines --- Pseudoscalar mesons*

Vertex: *Boxes* --- $\mathcal{L}^{(2)}_{\phi B}$, *Diamonds* --- $\mathcal{L}^{(4)}_{\phi B}$, *Solid dot* --- $\mathcal{L}^{(1)}_{\phi B}$, *circle-cross* --- $\mathcal{L}^{(2)}_{\phi B}$

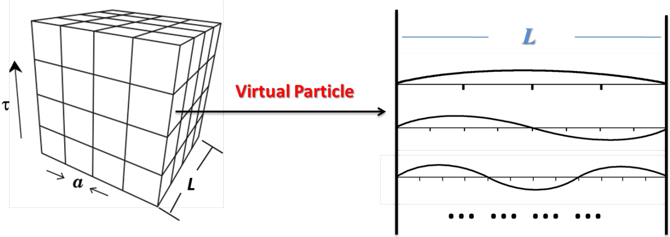
Octet baryon masses in infinite time-space

- Calculate **baryon self-energy** using dimensional regularization ($\overline{\mathrm{MS}}$)
- Subtract **PCB terms** with EOMS scheme

$$m_B^{\text{Inf.}} = m_0 + m_B^{(2)}(M_\phi) + m_B^{(3)}(M_\phi) + m_B^{(4)}(M_\phi).$$

Lattice Finite-Volume Corrections

☐ Physical picture of FVCs



- Momentum of virtual particle discretized

$$k_i = \frac{2\pi}{L} \cdot n_i, \quad (i = 0, 1, 2, 3) \quad \Longrightarrow \quad \int_{-\infty}^{\infty} dk \sim \sum_{n=-\infty}^{\infty} \frac{2\pi}{L} \cdot n$$

- Definition of FVCs:

$$\Delta H_{\rm FVC}^{(b)} = \int \frac{dk_0}{2\pi} \cdot \left(\frac{1}{L^3} \sum_{\vec{k}} \Box - \int \frac{d\vec{k}}{(2\pi)^3} \Box \right) \qquad \text{with} \;\; \mathrm{L_{time}} \sim 5\mathrm{L_{space}}.$$

Continuum extrapolation of LQCD

- In principle, continuum extrapolation should be **firstly performed**
  - **LQCD:** makes a statement about the underlying fundamental continuum theory
  - **BChPT:** describes the continuum QCD and is not valid for $a \neq 0$
- But, the most LQCD and BChPT studies
  - Discretizsation effects of LQCD are assumed **small**, always taken as **systematic error or neglected**

Discretization effects should be studied in BChPT

- Use Symanzik effective action *K.Symanzik, NPB(1983)*

$$S_{\rm eff} = S_0^{\rm QCD} + aS_1 + a^2 S_2 + \cdots$$

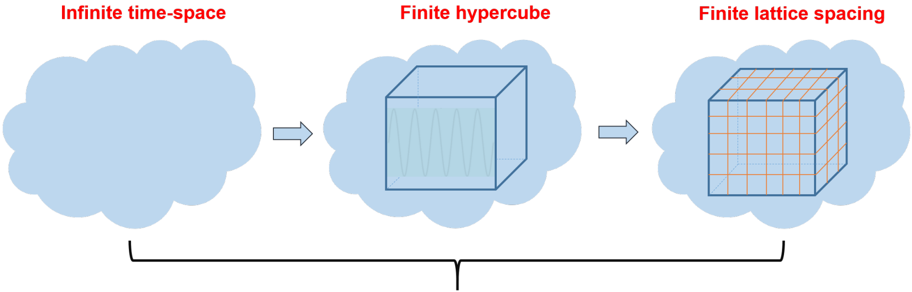
- Construct effective Lagrangians up to $\mathcal{O}(a^2)$

$$\mathcal{L}_a^{\rm eff} = \mathcal{L}^{\mathcal{O}(a)} + \mathcal{L}^{\mathcal{O}(am_q)} + \mathcal{L}^{\mathcal{O}(a^2)}.$$

- Calculate discretization effects of LQCD with the Wilson fermion

$$m_B^{(a)} = m_B^{\mathcal{O}(a)} + m_B^{\mathcal{O}(am_q)} + m_B^{\mathcal{O}(a^2)}.$$

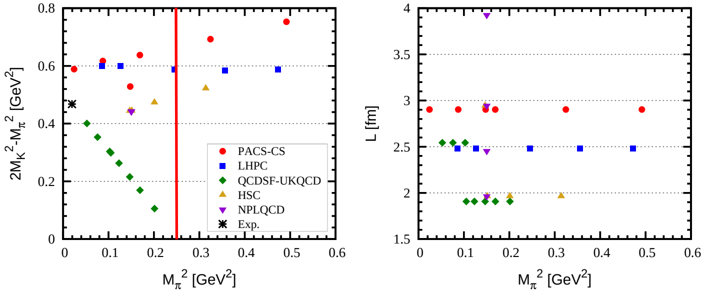
Octet baryon masses for LQCD



$$m_B^{\mathrm{LQCD}} = m_0 + m_B^{(2)}(M_\phi) + m_B^{(3)}(M_\phi) + m_B^{(4)}(M_\phi).$$

Numerical Details

- Fitting data: **LQCD results (11-sets)** + **Exp. values**
  - PACS-CS, LHPC, QCDSF-UKQCD, HSC, NPLQCD
    - Lattice data with $M_\pi < 500$ MeV
      - ☞ reduce the higher order contributions of chiral expansions
    - Lattice data with $M_\phi L > 4$
      - ☞ minimize finite-volume effects of LQCD
  - **Fitting points:** $44(\mathrm{LQCD}) + 4(\mathrm{Exp.}) = 48$



Results and Discussion

Octet baryon masses in EOMS BChPT

☞ Assuming: **virtual decuplet effects can be absorbed by LECs.**

- **Fitting methods**

	Fitting formula	Free parameters	
NLO	$m_0 + m_B^{(2)}$	m_0, b_0, b_D, b_F	**4**
NNLO	$m_0 + m_B^{(2)} + m_B^{(3)}$	m_0, b_0, b_D, b_F	**4**
N^3LO	$m_0 + m_B^{(2)} + m_B^{(3)} + m_B^{(4)}$	$m_0, b_0, b_D, b_F, b_i, d_j$	**19**

- **Other parameters**
  - $L^r_{4,5,6,7,8}$, *J. Bijnens et al., NPB(2012), with $\mu = 1$ GeV*
  - $F_0 = 0.0871$ GeV, *G. Amoros et al., NPB(2001)*
  - $D = 0.80$, $F = 0.46$

Best fitting results

	NLO	NNLO	N^3LO
m_0 [MeV]	$900(6)$	$767(6)$	**$880(22)$**
b_0 [GeV^{-1}]	$-0.273(6)$	$-0.886(5)$	$-0.609(19)$
b_D [GeV^{-1}]	$0.0506(17)$	$0.0482(17)$	$0.225(34)$
b_F [GeV^{-1}]	$-0.179(1)$	$-0.514(1)$	$-0.404(27)$
b_1 [GeV^{-1}]	--	--	$0.550(44)$
b_2 [GeV^{-1}]	--	--	$-0.706(99)$
b_3 [GeV^{-1}]	--	--	$-0.674(115)$
b_4 [GeV^{-1}]	--	--	$-0.843(81)$
b_5 [GeV^{-2}]	--	--	$-0.555(144)$
b_6 [GeV^{-2}]	--	--	$0.160(95)$
b_7 [GeV^{-2}]	--	--	$1.98(18)$
b_8 [GeV^{-2}]	--	--	$0.473(65)$
d_1 [GeV^{-3}]	--	--	$0.0340(143)$
d_2 [GeV^{-3}]	--	--	$0.296(53)$
d_3 [GeV^{-3}]	--	--	$0.0431(304)$
d_4 [GeV^{-3}]	--	--	$0.234(67)$
d_5 [GeV^{-3}]	--	--	$-0.328(60)$
d_7 [GeV^{-3}]	--	--	$-0.0358(269)$
d_8 [GeV^{-3}]	--	--	$-0.107(32)$
χ^2/d.o.f.	11.8	8.6	1.0

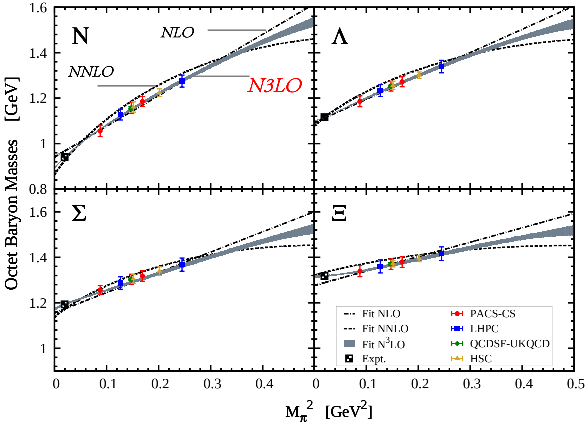
Table: Values of the LECs.

- EOMS-BChPT shows a clear improvement order by order
- Different lattice QCD calculations are consistent with each other
- Values of LECs from EOMS-N^3LO look very natural
- $m_0 = 880$ MeV consistent with the SU(2)-BChPT results.

  M. Procura et al.,PRD(2003,2006)

  L. Alvarez-Ruso et al., PRD(2013)
- **Neglecting finite-volume corrections would lead to $\chi^2/\mathbf{d.o.f.} = \mathbf{1.9}$.**

Chiral extrapolation



- **NLO fitting** linear and can not describe the experimental value
- **NNLO fitting** more curved and can not well describe lattice data
- **N^{3}LO fitting** can give a good description of LQCD and Exp. data, confirm the linear dependence of the lattice data on M_π^2

Up to now ...

☐ Multi-extrapolation of LQCD

$m_{u/d}^{\text{Lat.}}$	Chiral extrapolation $\Longrightarrow$	$m_{u/d}^{\text{Phys.}}$	✓
L	Finite−volume corrections $\Longrightarrow$	∞	✓
a	Continuum extrapolation $\Longrightarrow$	0	

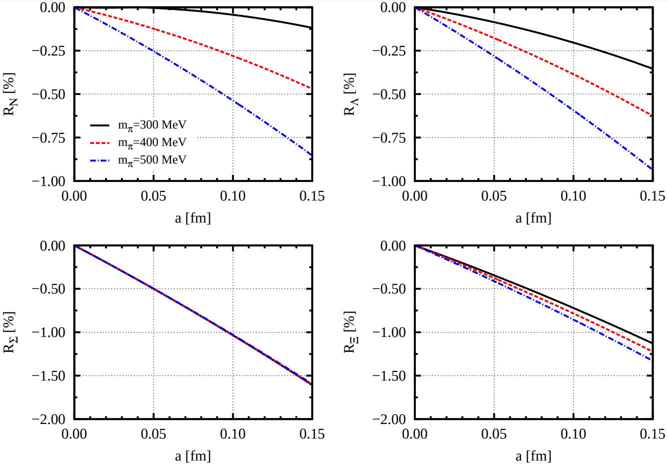
- **Including FVCs and discretization effects self-consistently** in the octet baryon masses

$$m_B = m_0 + m_B^{(2)} + m_B^{(3)} + m_B^{(4)} + m_B^{(a)}.$$

  - 19 LECs + **4 new LECs** (related to lattice spacing)

- Fitting the LQCD results obtained with $\mathcal{O}(a)$-improved Wilson action
  - **10 sets:** PACS-CS, QCDSF-UKQCD, HSC, NPLQCD

Evolution of discretization effects with a and m_π



$$R_B = \frac{m_B^{(a)}}{m_B}$$

- Pion mass m_π fixed:

$$a \Uparrow \quad \sim \quad m_B^{(a)} \Uparrow$$

- Lattice spacing a fixed:

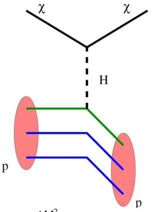
$$m_\pi \Uparrow \quad \sim \quad m_B^{(a)} \Uparrow$$

- Discretization effects on baryon masses **do not exceed 2% for $a = 0.15$ fm**
- **Consistent** with early LQCD studies *S. Durr et al., Phys. Rev. D79, (2009) 014501.*
- Up to $\mathcal{O}(a^2)$, discretization effects are **small** and can be safely **ignored**

Pion- and strangeness-octet baryon sigma terms

Nucleon-sigma term

- Related to chiral quark condensate $\langle \bar{q}q \rangle$
- Understand the **composition of the nucleon**
- Key input for direct **dark matter** searches
- **Strangeness-nucleon sigma term: $0 \sim 300$ MeV**



Feynman-Hellmann Theorem

$$\boldsymbol{\sigma_{\pi B}} = m_l \langle B(p)|\bar{u}u + \bar{d}d|B(p)\rangle \equiv \boldsymbol{m_l \frac{\partial m_B}{\partial m_l}},$$
$$\boldsymbol{\sigma_{sB}} = m_s \langle B(p)|\bar{s}s|B(p)\rangle \equiv \boldsymbol{m_s \frac{\partial m_B}{\partial m_s}}.$$

Three key factors to accurately predict baryon sigma terms

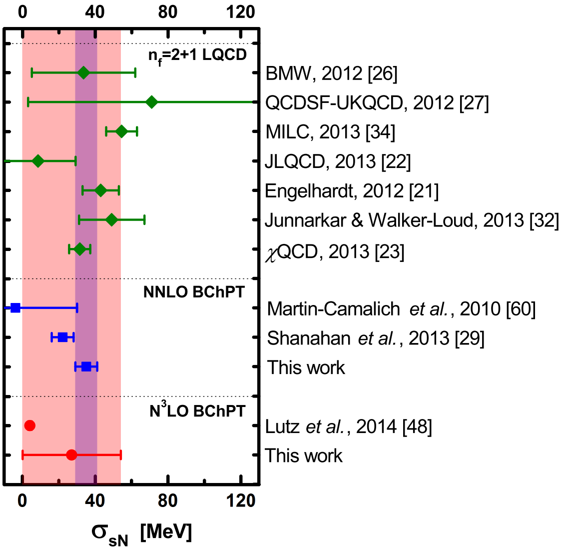
- **Effects of lattice scale setting:** mass independent vs. mass dependent
- **Strong isospin breaking effects:** better constrain the values of LECs
- **Chiral expansion truncations:** systematic uncertainties of sigma terms

Octet baryon sigma terms from N^3LO BChPT

	MIS		MDS
	a fixed	a free	
$\sigma_{\pi N}$	55(1)(4)	54(1)	51(2)
$\sigma_{\pi\Lambda}$	32(1)(2)	32(1)	30(2)
$\sigma_{\pi\Sigma}$	34(1)(3)	33(1)	37(2)
$\sigma_{\pi\Xi}$	16(1)(2)	18(2)	15(3)
σ_{sN}	27(27)(4)	23(19)	26(21)
$\sigma_{s\Lambda}$	185(24)(17)	192(15)	168(14)
$\sigma_{s\Sigma}$	210(26)(42)	216(16)	252(15)
$\sigma_{s\Xi}$	333(25)(13)	346(15)	340(13)

- All three scale setting methods yield similar baryon sigma terms.

σ_{sN}: comparison with earlier studies



- Our result consistent with the latest LQCD and NNLO BChPT results
- NNLO σ_{sN} result has a much smaller uncertainty compared to the N^3LO
- Call for more LQCD data of octet baryon masses, especially with the smaller strange quark masses

Summary

- We have systematically studied the LQCD octet baryon masses with the EOMS BChPT up to N^3LO

- Finite-volume and disretizaion effects on the lattice data are taken into account self-consistently

- Through simultaneously fitting "all" the current LQCD data:
  - Covariant BChPT shows a clear improvement order by order
  - LQCD results are consistent with each other, though their setups are quite different
  - Up to $\mathcal{O}(a^2)$, the discretization effects on the LQCD baryon masses are shown to be small and can be safely ignored

- An accurate determination of the octet baryon sigma terms via the Feynman-Hellmann theorem.

Thank you!