Toward an effective field theory (EFT) approach in energy density functional theory

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Assemblee Generale

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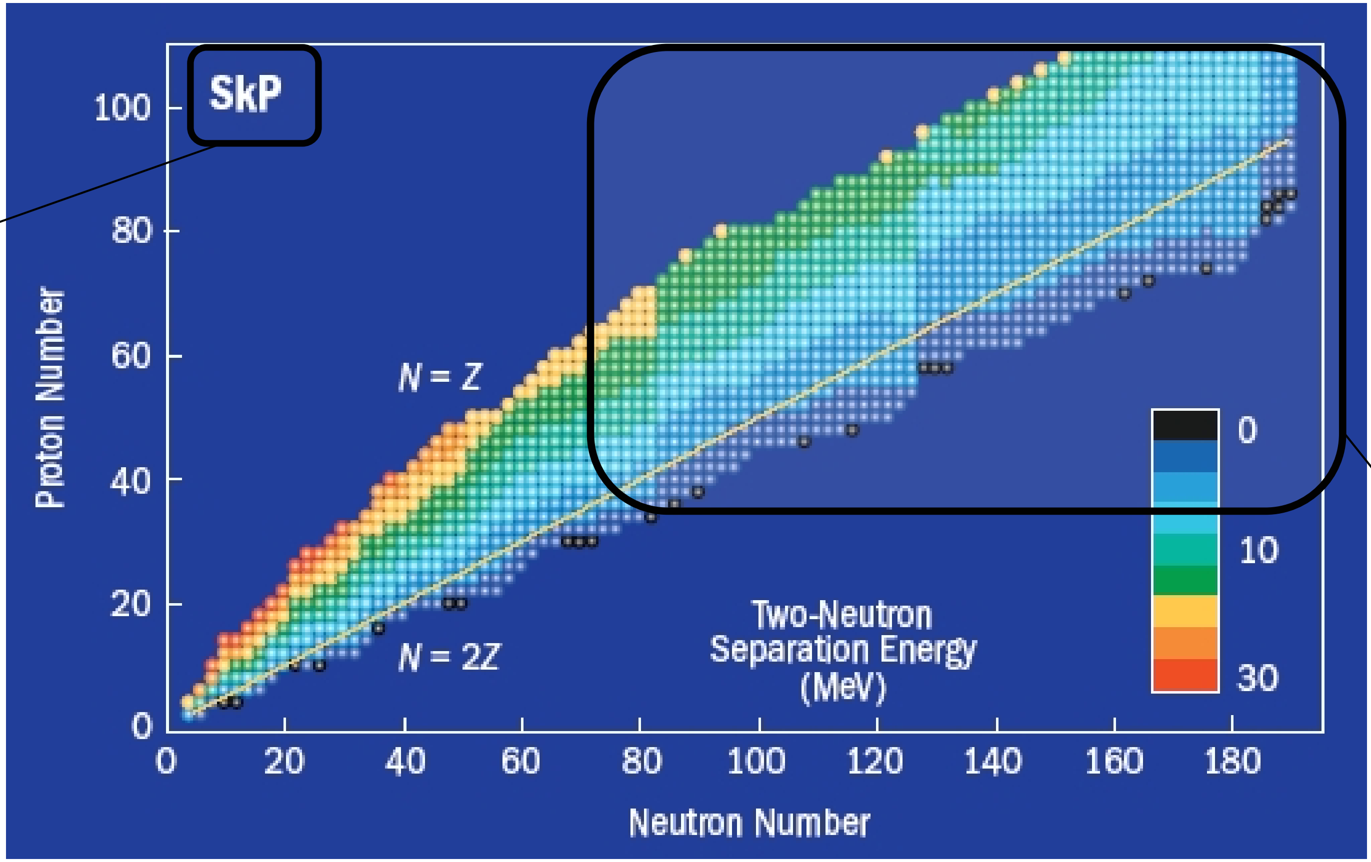
Motivation



Skyrme-type interaction works o.k. (able to do the fitting in EDF framework)

No way to get with ab-initio!

Need to think about other expansion (than on NN d.o.f.).

Present status of EDF

Works o.k., but:

1. What's the form of $V_{eff}^{Skp,Sly5,Gongy...}$?
2. ***No power counting***, if stop at mean field.

Outcome:

- Include more parameters won't necessarily help.

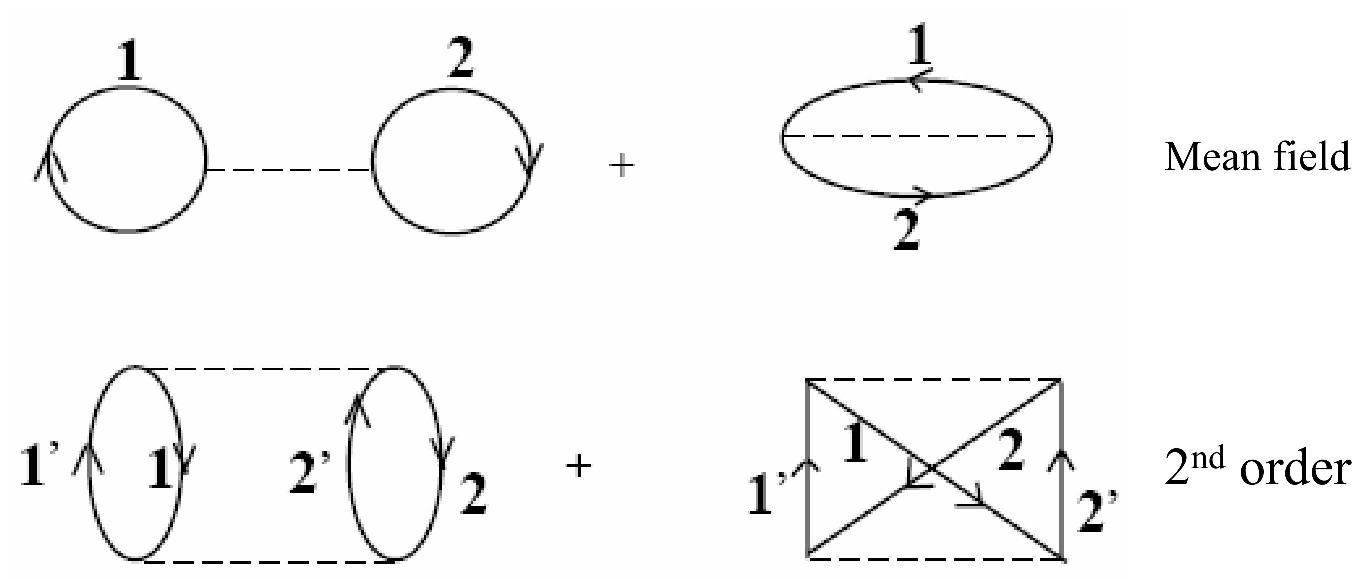
→ Limited predictive power.

Reason: Maybe the correct theory has a structure where some terms should appears (be fitted) at higher order.

→ Need to go beyond mean field to perform the test.

2^{nd} order correction

$$\frac{E}{A} = \underbrace{\frac{E^{(0)}}{A}}_{\text{mean field}} + \underbrace{\frac{E^{(2)}}{A}}_{2^{nd}\ \text{order}} + \ldots$$



2nd order results of nuclear matter

(C.J. Yang, M. Grasso, X. Roca-Maza, G. Colo, and K. Moghrabi, arXiv:1604.06278)
In agreement with N.Kaiser, J. Phys. G 42,095111(2015)

$$\frac{\Delta E^{(2)}_{sym(l=0)}}{A} = -\frac{mk_F^4}{110880\hbar^2\pi^4}\left\{\begin{array}{c}\left[\begin{array}{c}-6534+1188ln[2]+3564\lambda-19602\lambda^3-5940\lambda^5\\ +(1782-20790\lambda^4)ln[\frac{\lambda-1}{\lambda+1}]\\ +(24948\lambda^5-5940\lambda^7)ln[\frac{\lambda^2-1}{\lambda^2}]\end{array}\right]\widetilde{T}_{03}^2\\ +\left[\begin{array}{c}-\quad 14696+2112ln[2]+5280\lambda-2860\lambda^3\\ -48840\lambda^5-18480\lambda^7+(2640-55440\lambda^6)ln[\frac{\lambda-1}{\lambda+1}]\\ +(71280\lambda^7-18480\lambda^9)ln[\frac{\lambda^2-1}{\lambda^2}]\end{array}\right]k_F^2\widetilde{T}_{03}\widetilde{T}_1\\ +\left[\begin{array}{c}-9886+1128ln[2]+\ 2520\lambda+147\lambda^3-3654\lambda^5\\ -35280\lambda^7-15120\lambda^9+(1260-41580\lambda^8)ln[\frac{\lambda-1}{\lambda+1}]\\ +(55440\lambda^9-15120\lambda^{11})ln[\frac{\lambda^2-1}{\lambda^2}]\end{array}\right]k_F^4\widetilde{T}_1^2\end{array}\right\}$$

Diverge as Λ^5

$$\frac{\Delta E^{(2)}_{sym(l=1)}}{A} = -\frac{mk_F^8}{73920\hbar^2\pi^4}\left\{\left[\begin{array}{c}-1033+156ln[2]+420\lambda+140\lambda^3-840\lambda^5\\ -5880\lambda^7-2520\lambda^9+(-210+6930\lambda^8)ln[\frac{\lambda-1}{\lambda+1}]\\ +(9240\lambda^9-2520\lambda^{11})ln[\frac{\lambda^2-1}{\lambda^2}]\end{array}\right]\widetilde{T}_2^2\right\},$$

$$\frac{\Delta E^{(2)}_{neutr(l=0)}}{A} = -\frac{mk_{F_N}^4}{166320\hbar^2\pi^4}\left\{\begin{array}{c}\left[\begin{array}{c}-6534+1188ln[2]+3564\lambda-19602\lambda^3-5940\lambda^5\\ +(1782-20790\lambda^4)ln[\frac{\lambda-1}{\lambda+1}]\\ +(24948\lambda^5-5940\lambda^7)ln[\frac{\lambda^2-1}{\lambda^2}]\end{array}\right]T_{03}^2\\ +\left[\begin{array}{c}-\quad 14696+2112ln[2]+5280\lambda-2860\lambda^3\\ -48840\lambda^5-18480\lambda^7+(2640-55440\lambda^6)ln[\frac{\lambda-1}{\lambda+1}]\\ +(71280\lambda^7-18480\lambda^9)ln[\frac{\lambda^2-1}{\lambda^2}]\end{array}\right]k_{F_N}^2T_{03}T_1\\ +\left[\begin{array}{c}-9886+1128ln[2]+\ 2520\lambda+147\lambda^3-3654\lambda^5\\ -35280\lambda^7-15120\lambda^9+(1260-41580\lambda^8)ln[\frac{\lambda-1}{\lambda+1}]\\ +(55440\lambda^9-15120\lambda^{11})ln[\frac{\lambda^2-1}{\lambda^2}]\end{array}\right]k_{F_N}^4T_1^2\end{array}\right\}$$

Diverge as Λ^5

$$\frac{\Delta E^{(2)}_{neutr(l=1)}}{A} = -\frac{mk_{F_N}^8}{110880\hbar^2\pi^4}\left\{\left[\begin{array}{c}-1033+156ln[2]+420\lambda+140\lambda^3-840\lambda^5\\ -5880\lambda^7-2520\lambda^9+(-210+6930\lambda^8)ln[\frac{\lambda-1}{\lambda+1}]\\ +(9240\lambda^9-2520\lambda^{11})ln[\frac{\lambda^2-1}{\lambda^2}]\end{array}\right]T_2^2\right\},$$

Application:
Neutron matter at very low- ρ

(C.J. Yang, M. Grasso, D Lacroix, arXiv:1604.06587)

Lee & Yang formula (1957) describes the dilute system.

(expansion in $k_N a$)

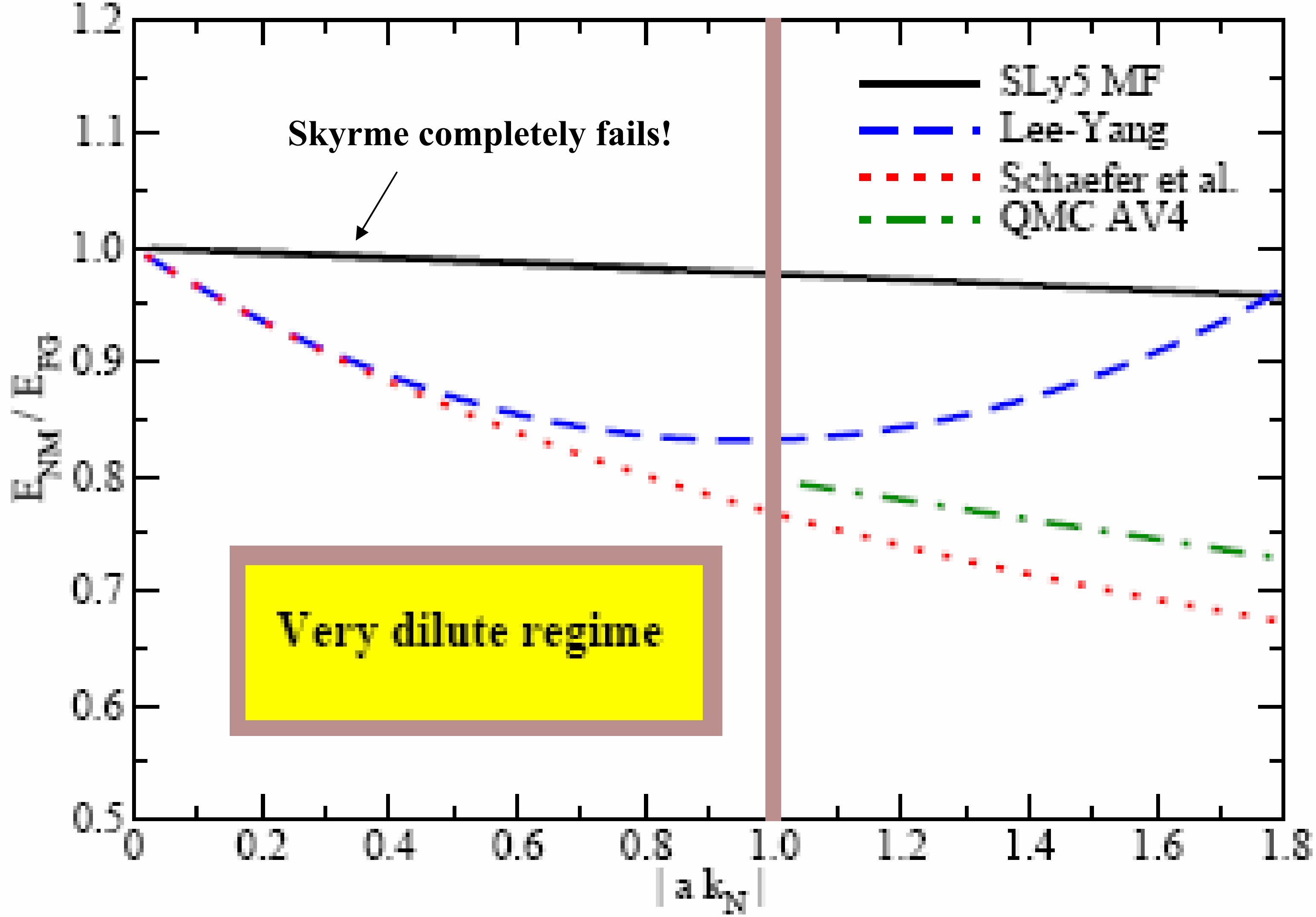
$$\frac{E_{NM}}{A} = \frac{\hbar^2 k_N^2}{2m}\left[\underbrace{\frac{3}{5}}_{K.E.} + \underbrace{\frac{2}{3\pi}(k_N a)}_{\textit{fixed}\ \text{to}\ t_0\ \text{term}} + \underbrace{\frac{4}{35}(11-2\ln 2)(k_N a)^2}_{\textit{automatically}\ \text{recover in}\ 2^{\text{nd}}\ \text{of}\ t_0} + \underbrace{O(k_N^3)}_{\textit{higher}\ \text{order}}\right]$$

• The 2nd order EoS automatically recover the $(11\text{-}2\ln 2)(k_N a)^3$ term!

• If take physical value of a=-18.9 fm, then *impossible* to fit pure neutron matter EoS outside region $k_N a \ll 1$ [adding more terms (e.g., t_1, t_2, t_3 in Skyrme) won't help].

=> $(k_N a)$ needs to be re-summed. (Steele (2000), Schafer (2005), Kaiser (2011))

$$\frac{E_{NM}}{N} = \frac{\hbar^2 k_N^2}{2m}\left[\frac{3}{5} + \frac{2}{3\pi}\frac{k_N a}{1-6k_N a(11-2\ln 2)/(35\pi)}\right]$$



Skyrme completely fails!
SLy5 MF
Lee-Yang
Schaefer et al.
QMC AV4
Very dilute regime
E_NM / E_FG
| a k_N |
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8

YGLO: Partially resumed functional

(**Yang-Marcella-Lacroix Orsay**, C.J. Yang, M. Grasso, D Lacroix, arXiv:1604.06587)

$$V_{YGLO} = \frac{B_\beta}{1 - R_\beta \rho^{1/3} + \underbrace{C_\beta \rho^{2/3}}_{\text{higher order in L\&Y to be resumed*}}} + \underbrace{D_\beta \rho^{2/3}}_{velocity-dep\ \text{term*}} + \underbrace{F_\beta \rho^{\alpha}}_{3^+ - body}$$

B_β, R_β are fixed to reproduce first two term in Lee & Yang.

$$\Rightarrow B_\beta = 2\pi \frac{\hbar^2}{\mathrm{m}} \frac{\nu - 1}{\nu} a_\beta,\ R_\beta = \frac{6}{35\pi} \left(\frac{6\pi^2}{\nu} \right)^{1/3} (11 - 2\ln 2) a_\beta.$$

(degeneracy: $\nu = 2(4)$ for $\beta = \underbrace{0}_{\text{pure n}} \ (\underbrace{1}_{\text{sym}}))$

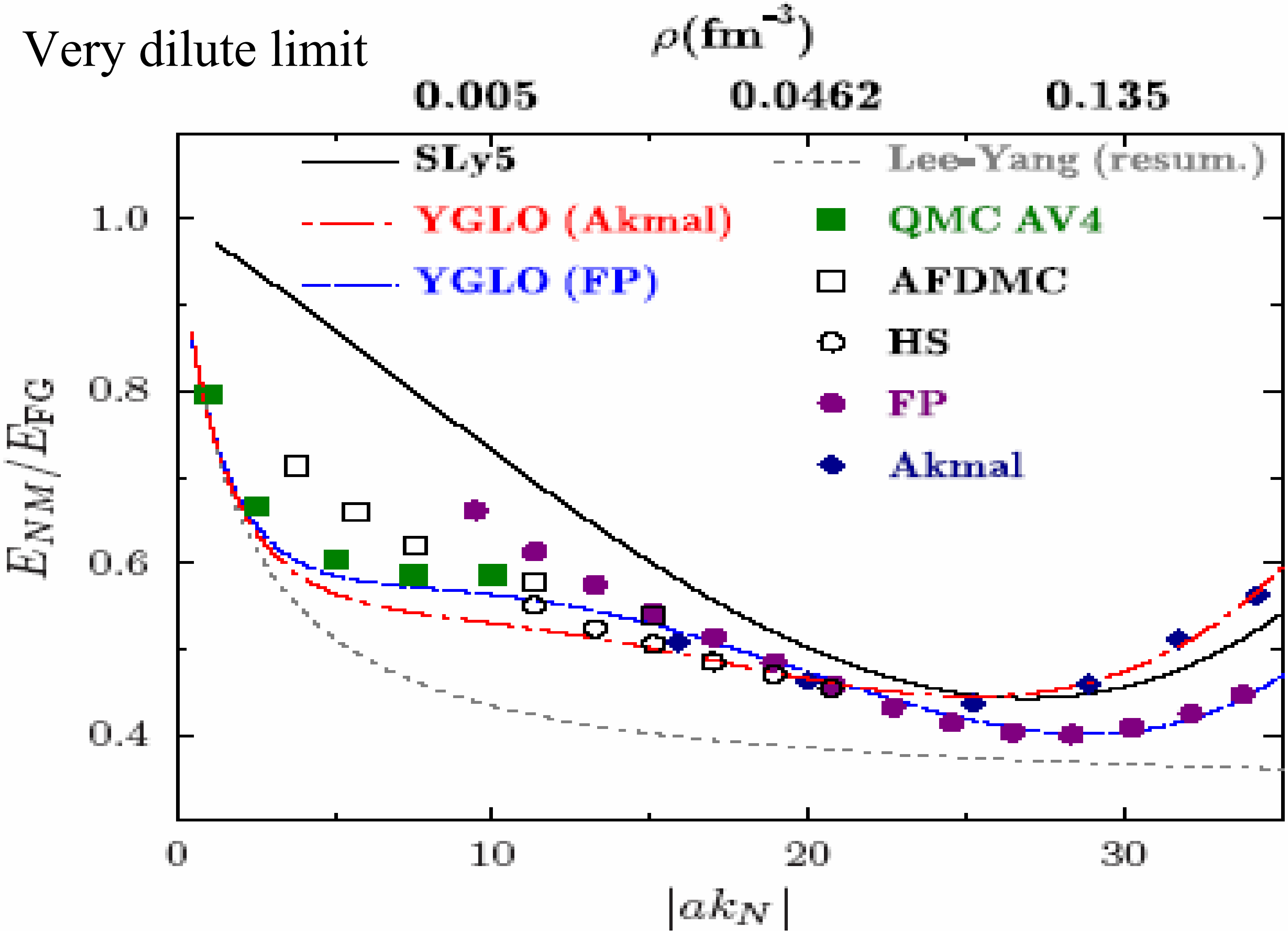
$$a_0 = -18.9\text{fm},\quad \underbrace{a_1 = -20\ \text{fm}}_{avg.\ \text{of}\ \mathrm{a_{nn}},\ \mathrm{a_{pp}},\ \mathrm{a_{np}}\ \text{in}\ {}^1S_0}.$$

$$\frac{E}{A} = KE_\beta + \frac{B_\beta \rho}{1 - R_\beta \rho^{1/3} + C_\beta \rho^{2/3}} + D_\beta \rho^{5/3} + F_\beta \rho^{\alpha+1}.$$

Four free parameters to be fitted.

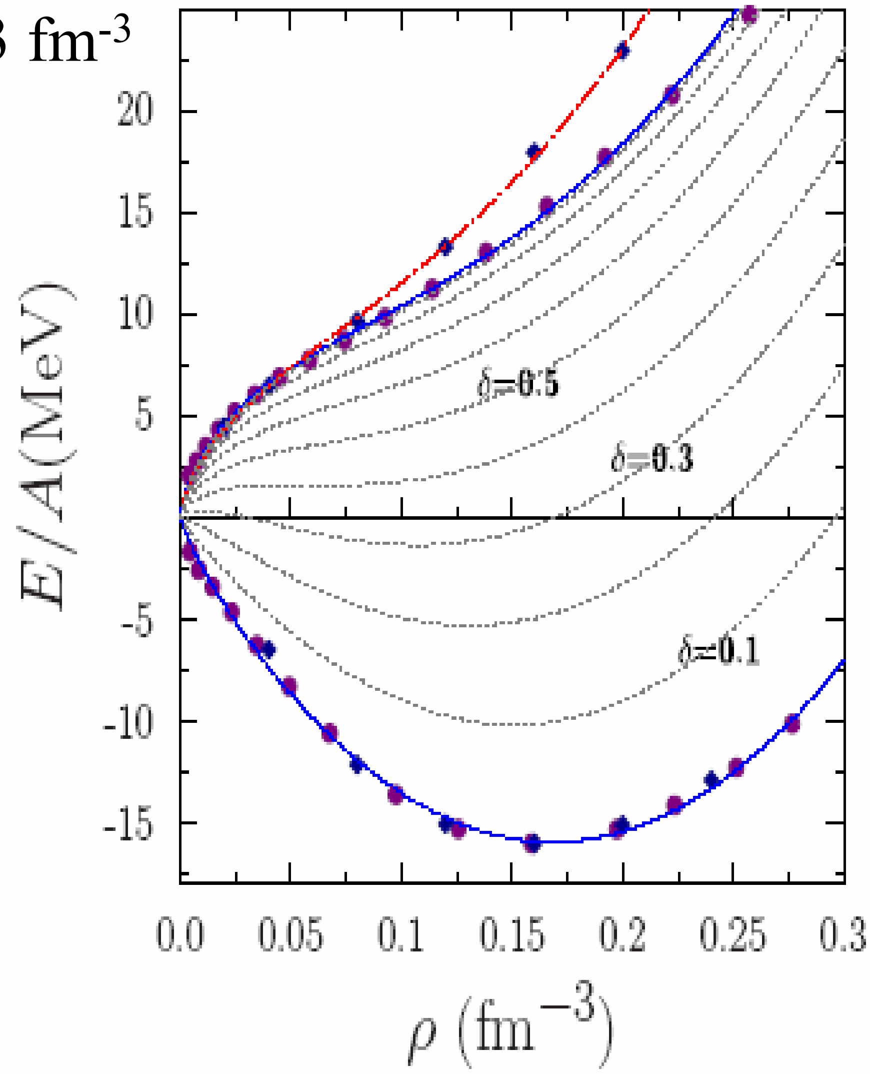
Results

Very dilute limit



FP: B. Friedman and V. Pandharipande, Nucl. Phys. A361,502 (1981).
Akmal: A. Akmal, V. R. Pandharipande, and D. G. Ravenhall, Phys. Rev. C 58, 1804 (1998).
HS: K. Hebeler and A. Schwenk Phys. Rev. **C 82**, 014314(2010).

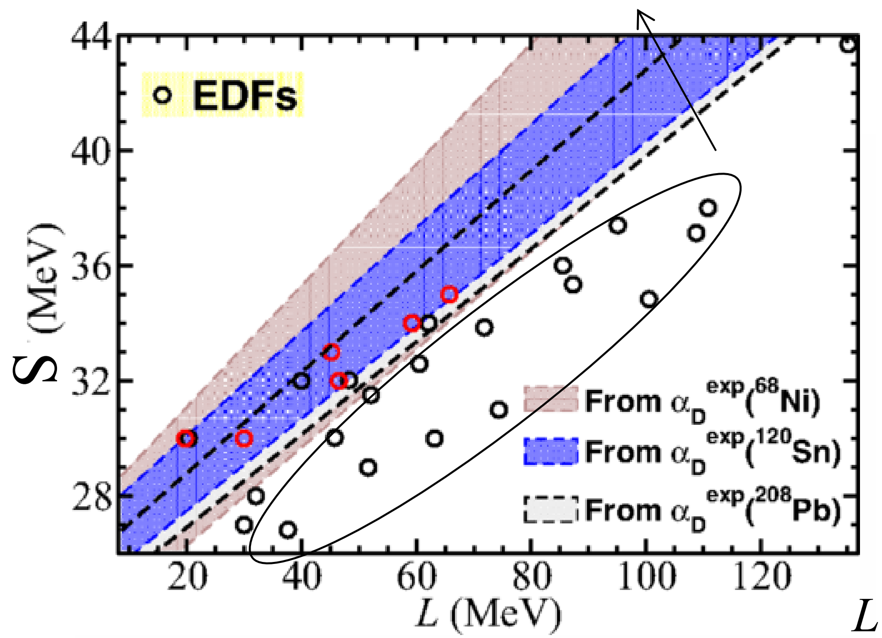
Up to ρ =0.3 fm^{-3}



Able to describe both sym and pure neutron matter EoS up to 2 ρ_0 very well with only 4 free parameters each.

Prediction: Asymmetric case

Before: Lots of models fail



$$L = 3\rho_0 (dS/d\rho)_{\rho=\rho_0}$$

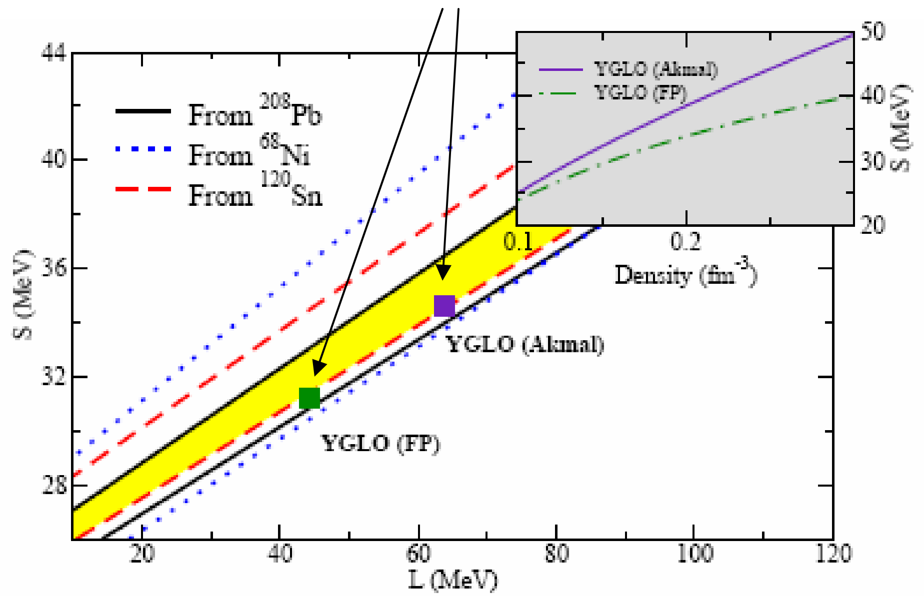
FIG. 4: Symmetry energy at saturation density as a function of its slope L. The black lines delimit the phenomenological area constrained by the experimental determination of the electric dipole polarizability in ^{208}Pb. The blue dotted lines delimit the area constrained by the same measurement in ^{68}Ni, and the red dashed lines refer to the measurement done in ^{120}Sn. The yellow area is the overlap. Inset: density dependence of the Symmetry energy for the two YGLO parametrizations of this work.

X. Roca-Maza, et al., (2015).

Prediction: Asymmetric case

Our result

Satisfies the experimental constraint.



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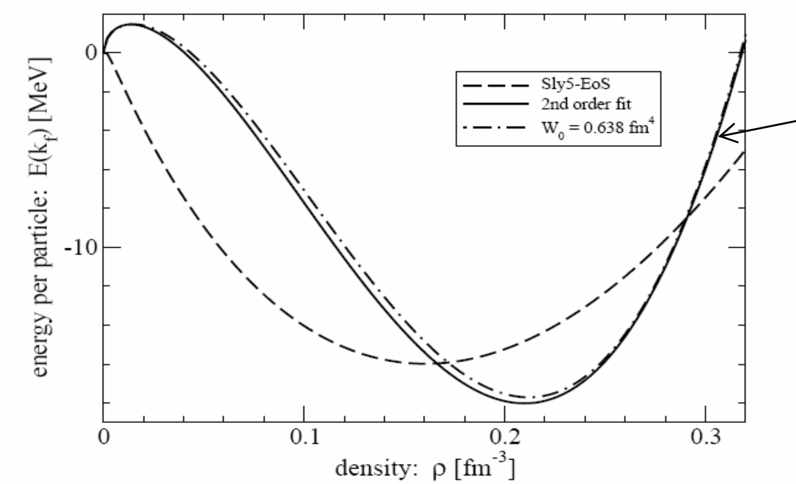
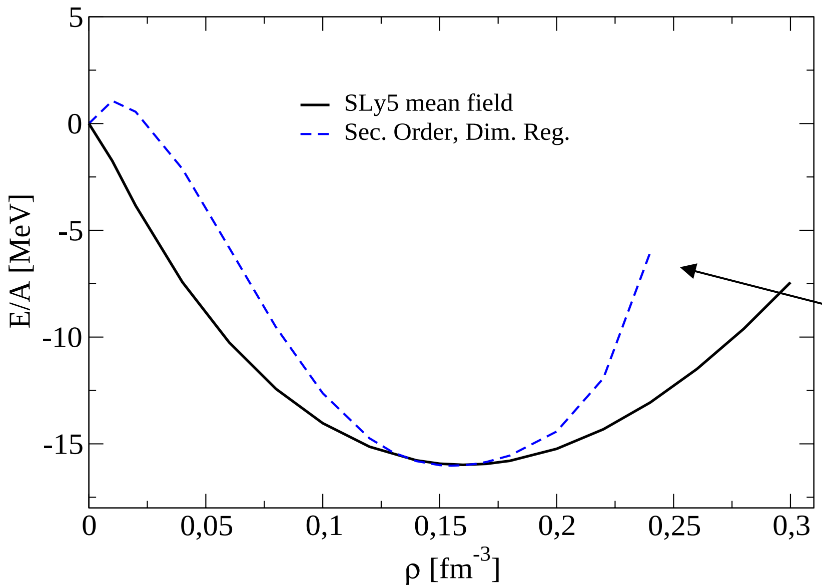
Alternative choice:
What if one treats all (Skyrme-like) terms perturbatively?

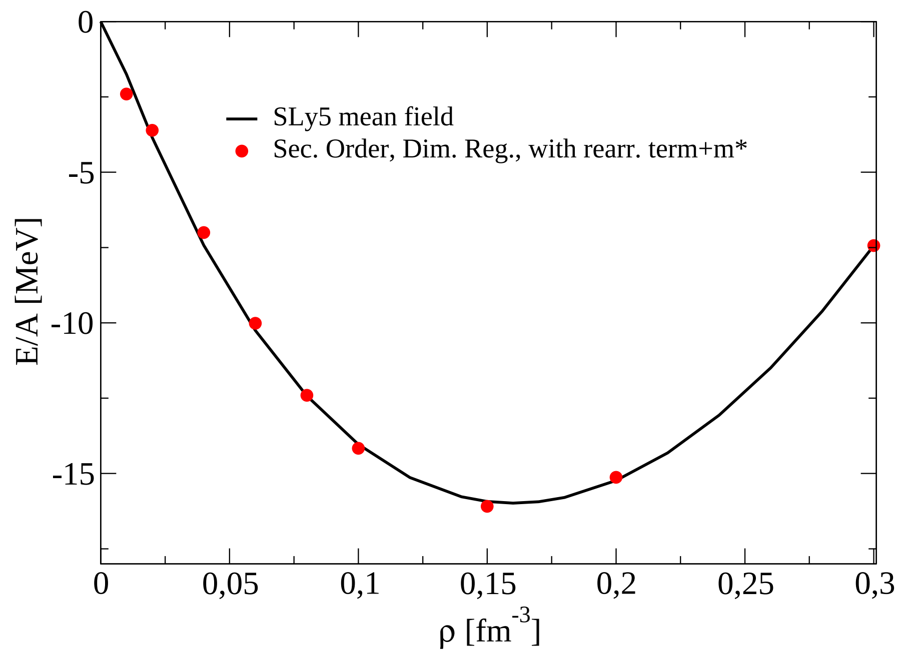
Effect of ρ -dep & rearrangement terms

(2nd order SNM, under Dimensional regularization)



Without density-dep.(t_3) term, Kaiser 2015





With density-dep., without (with) rearrangement term

(C.J. Yang, M. Grasso, X. Roca-Maza, G. Colo, and K. Moghrabi, arXiv:1604.06278)

Some lessons

- Non-perturbative treatment of some part (t_0) of the interaction is necessary to describe low ρ limit correctly.
- If ρ -dep term is treated perturbatively (Dyson series), its rearrangement term is important.

Merci!
Thank you!

Road map

Try to bridge EFT ideas/techniques to mean field (and beyond) within EDF framework.

Mean field with potential models (effective interaction).
(e.g., Skyrme-type)

2nd order corrections

Higher order corrections

Add new effective interactions?

What is the proper form of it?

Is the improvement systematic?

Renormalization-group analysis
+
power counting check

Goal: Systematic treatment of the interactions.

Interaction & mean field EoS

Interaction: Skyrme without spin-orbit

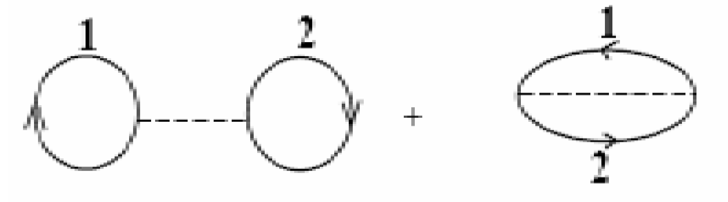
$$v = \underbrace{t_0(1+x_0P_\sigma)}_{S-wave\ \mathrm{O(0)}} + \frac{1}{2}\underbrace{t_1(1+x_1P_\sigma)(k'^2+k^2)}_{S-wave\ \mathrm{O(q^2)}} + \underbrace{t_2(1+x_2P_\sigma)\mathbf{k}'\cdot\mathbf{k}}_{p-wave\ \mathrm{O(q^2)}}$$

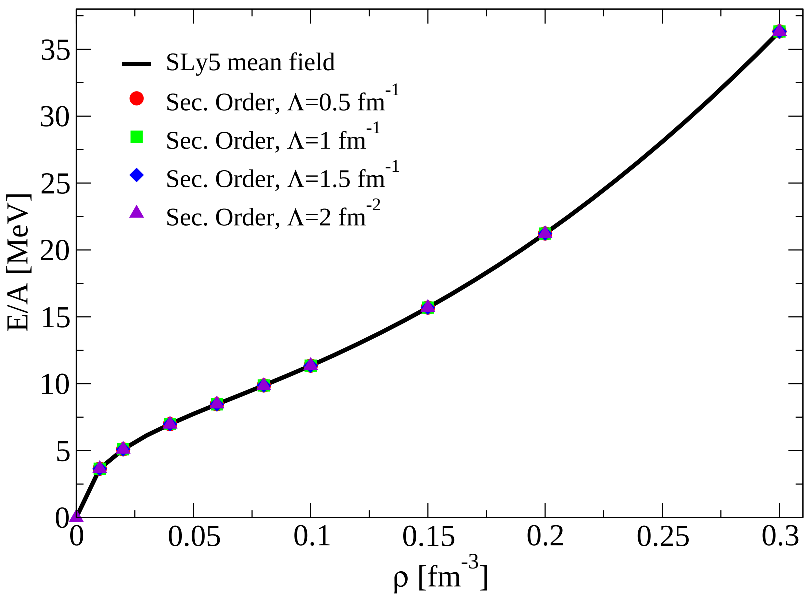
$$+\frac{1}{6}\underbrace{t_3(1+x_3P_\sigma)\rho^\alpha}_{s-wave,\ \text{higher body}}.$$

$$\mathrm{P}_\sigma = \frac{1}{2}(1+\sigma_1\cdot\sigma_2)$$

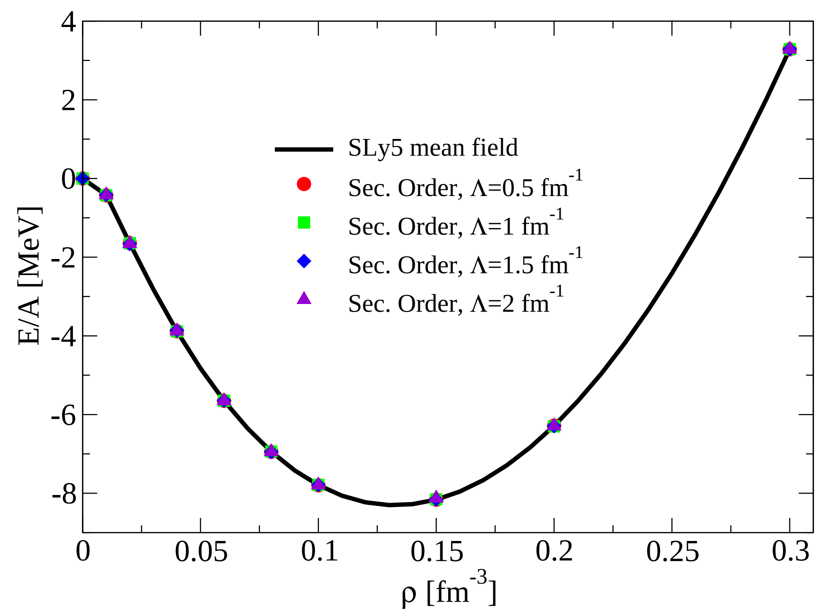
No pion! Like pionless EFT, except for the density-dependent term.

$$EoS: \quad \frac{E}{A} \propto \frac{1}{\rho}\int_0^{k_{F1}} d^3k_1 \int_0^{k_{F2}} d^3k_2 v$$





Pure neutron matter



Asymmetric matter (δ =0.5)

EFT v.s. Model

- Both establish the (effective) interaction by fitting a set of free parameters to data.

EFT

Assume a leading order (LO) interaction.

Consider QM correction (NLO, NNLO, etc…).

At each order, fit the free parameters (renormalization).

Establish power counting based on their importance, i.e., $\mathfrak{A} = \sum (E/\Lambda)^n$.

Model

Assume an interaction, which contains free parameters to describe the systems of interest.

Increase # of parameters, until:
I. Results are good enough.
II. It costs too much to be solved.