$NN\ ^1S_0$ Channel in Effective Field Theory

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Overview

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- ✓ Not a true bound state, but almost! Consequences of this on **nucleosynthesis** and the abundance of chemical elements.
- ✓ **Lattice QCD results:** $B_{^1S_0}\left[m_\pi = 800 \text{ MeV}\right] \approx 16 \text{ MeV}$ (NPLQCD, 2013). $B_{^1S_0}\left[m_\pi = 300 \text{ MeV}\right] \approx 9 \text{ MeV}$ (Yamazaki *et al.*, 2015).

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- ✓ Thanks to *ab–initio* calculations, one might build the bridge between **nuclear forces and nuclear structure**. We would like to derive the nuclear potential from first principles.
- ✓ Essential difficulty: QCD is highly **non–perturbative** in the nuclear regime, where the basic d.o.f.'s are (colorless) non–relativistic nucleons.
- ✓ In our desired EFT of QCD, one has **multiplicity of scales**: $M_{\rm QCD} \sim 4\pi f_\pi \sim m_\rho \sim 1$ GeV; $k_F \sim \frac{1}{R_{\rm rms}} \sim f_\pi \sim m_\pi \sim 100$ MeV; $\frac{B}{A} \sim \frac{k_F^2}{M_{\rm QCD}} \sim 10$ MeV. Daunting challenge?

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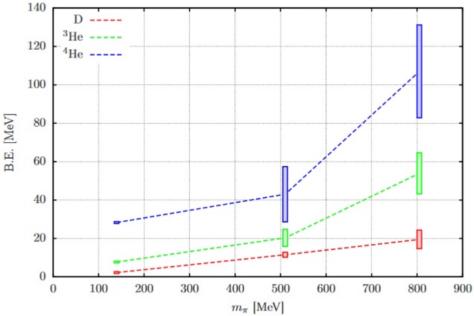

1] Lattice QCD. This *brute–force method* tries to solve numerically low–energy QCD on a discrete grid of points. Inconvenients: huge computational cost; large quark masses are needed as inputs. (On the left, results from the NPLQCD group and Yamazaki *et al.*)

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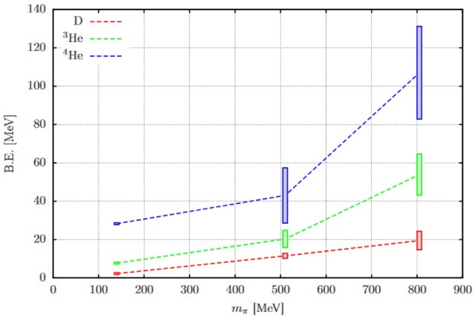

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2] Chiral EFT. This indirect method is based on the proposition of a chiral Lagrangian $\mathscr{L}_\chi$ which is the LE RGE of $\mathscr{L}_{\rm QCD}$. The connection is ensured by Chiral Symmetry and its breaking.

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- ✓ $g_i(\Lambda)$ are dim–less coefficients depending on the full theory, whose running is determined by *RGI*.

χPT

- ✓ Lightest pseudoscalar mesons $\simeq$ GB. *E.g.*, $\left(\pi^-,\pi^0,\pi^+\right)$ show SχSB $\left[SU\left(2\right)_L\otimes SU\left(2\right)_R\to SU\left(2\right)_V\right]$ in LE QCD.

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$$\mathscr{L}_{\pi\pi} = \mathscr{L}_{\pi\pi}^{(2)} + \cdots = \frac{f_\pi^2}{4}\mathrm{Tr}\left[\partial_\mu U^\dagger \partial^\mu U + m_\pi^2\left(U + U^\dagger\right)\right] + \cdots,$$
being $U \equiv \exp\left(\frac{i}{f_\pi}\Pi\right)$ with $\Pi = \boldsymbol{\tau}\cdot\boldsymbol{\pi} = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$.

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- ✓ *Infrared enhancement*: propagators are not $\sim Q^{-1}$, but $\sim \left(Q^2/M_N\right)^{-1}$ instead.

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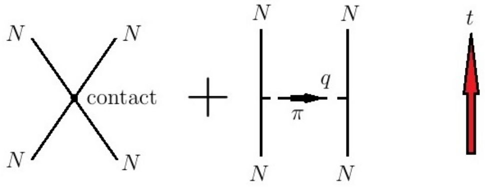
- ✓ $V_{\mathrm{LO}}(\vec{q}) = [C_S + (\vec{\sigma}_1\cdot\vec{\sigma}_2)C_T] - \frac{g_A^2}{4f_\pi^2}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\frac{(\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})}{\vec{q}^2+m_\pi^2} \sim Q^0$.



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$$+ \frac{g_A^2 m_\pi^2}{48\pi f_\pi^2}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left\{S_{12}(\hat{r})\left[1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right] + \vec{\sigma}_1\cdot\vec{\sigma}_2\right\}\frac{e^{-m_\pi r}}{r}$$

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$S_{12} = \sigma_1^i T^{ij} \sigma_2^j$
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only contributes for
$|L_f - L_i| = 2$.

- ✓ $T^{33} = \left(3\cos^2\theta - 1\right) \propto \mathscr{Y}_{\mathbf{2}}^{0}$
- ✓ $T^{31} = 3\cos\theta\sin\theta\sin\phi \propto \left(\mathscr{Y}_{\mathbf{2}}^{-1} + \mathscr{Y}_{\mathbf{2}}^{+1}\right)$
- ✓ $\cdots$

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$$\left.\begin{aligned}(\boldsymbol\tau_1\cdot\boldsymbol\tau_2)_{^1S_0} &= [2I(I+1)-3]_{I=1} = +1\\ (\vec\sigma_1\cdot\vec\sigma_2)_{^1S_0} &= [2S(S+1)-3]_{S=0} = -3\end{aligned}\right\}\Longrightarrow \boxed{U_{\rm LO}^{(^1S_0)}(\vec r) = M_N C_0\,\delta(\vec r) - \frac{m_\pi^2 e^{-m_\pi r}}{\Lambda_{NN} r}}$$

with $C_0 \equiv C_S - 3C_T + \frac{g_A^2}{4f_\pi^2}$ and $\Lambda_{NN} \equiv \frac{16\pi f_\pi^2}{g_A^2 M_N} \approx 300$ MeV (KSW'98).

$$\boxed{\langle \vec p\,'|V_{\rm LO}^{(^1S_0)}|\vec p\rangle = C_0 - \frac{4\pi}{M_N}\frac{m_\pi^2}{\Lambda_{NN}}\frac{1}{\vec q^{\,2}+m_\pi^2} \equiv \langle\vec p\,'|V_{\rm S}+V_{\rm L}|\vec p\rangle}$$

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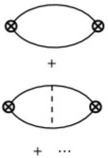

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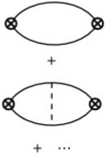

✓ $C_0\left(\Lambda\right)$ cannot absorb the divergence $\propto \frac{m_\pi^2}{\Lambda_{NN}}\log\Lambda$ (KSW'96).

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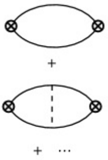
$$\overset{\Lambda\to\infty}{\sim} \frac{1}{\frac{1}{C_0(\Lambda)} + \frac{M_N}{4\pi}\left(\Lambda + \frac{m_\pi^2}{\Lambda_{NN}} \log\Lambda + \text{finite}\right)}$$



- ✓ $C_0\left(\Lambda\right)$ cannot absorb the divergence $\propto \frac{m_\pi^2}{\Lambda_{NN}} \log\Lambda$ (KSW'96).
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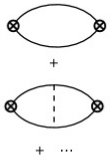

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- ✓ $\frac{1}{C_0} \mapsto \frac{1}{C_0 + D_2 m_\pi^2} = \frac{1}{C_0} - \frac{D_2}{C_0^2} m_\pi^2 + \frac{D_2^2}{C_0^3} m_\pi^4 + \cdots \Longrightarrow \left\{\begin{matrix} C_0\left(\Lambda\right) \overset{\Lambda\to\infty}{\sim} -\frac{4\pi}{M_N}\frac{1}{\Lambda}; \\ D_2\left(\Lambda\right) \overset{\Lambda\to\infty}{\sim} \frac{4\pi}{M_N \Lambda_{NN}}\frac{\log\Lambda}{\Lambda^2}.\end{matrix}\right.$

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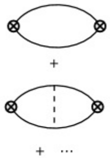

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- ✓ $\frac{1}{C_0} \mapsto \frac{1}{C_0+D_2m_\pi^2} = \frac{1}{C_0} - \frac{D_2}{C_0^2}m_\pi^2 + \frac{D_2^2}{C_0^3}m_\pi^4 + \cdots \Longrightarrow \begin{cases} C_0(\Lambda) \overset{\Lambda\to\infty}{\sim} -\frac{4\pi}{M_N}\frac{1}{\Lambda}; \\ D_2(\Lambda) \overset{\Lambda\to\infty}{\sim} \frac{4\pi}{M_N\Lambda_{NN}}\frac{\log\Lambda}{\Lambda^2}. \end{cases}$
- ✓ *Extra* residual cutoff dependence: $\frac{D_2^2(\Lambda)}{C_0^3(\Lambda)}m_\pi^4 + \cdots \overset{\Lambda\to\infty}{\sim} -\frac{M_N}{4\pi}\frac{m_\pi^4}{\Lambda_{NN}^2}\frac{\log^2\Lambda}{\Lambda} + \cdots$

$$V_S^{(\mathrm{reg})}(r) = \left(C_0(R) + D_2(R)\, m_\pi^2\right) \frac{\delta(r-R)}{4\pi R^2} \quad \mapsto \quad \tilde{V}_S^{(\mathrm{reg})}(r) = \frac{\delta(r-R)/4\pi R^2}{E_0(R)+F_2(R) m_\pi^2}$$

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With this, solve RSE's with the inputs $a_{^1S_0}[m_\pi = 138 \text{ MeV}] = -23.7 \text{ fm}$ and $a_{^1S_0}[m_\pi = 450 \text{ MeV}] = 1.8 \text{ fm}$ (Orginos *et al.*, 2015), using
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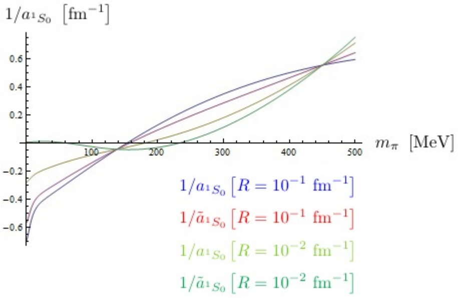

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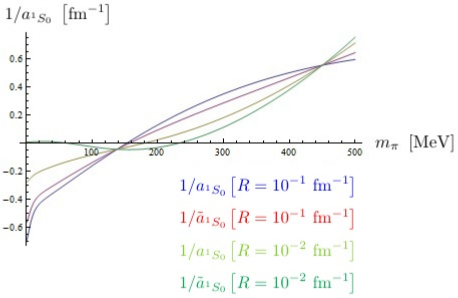
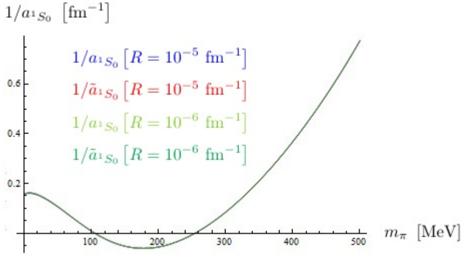
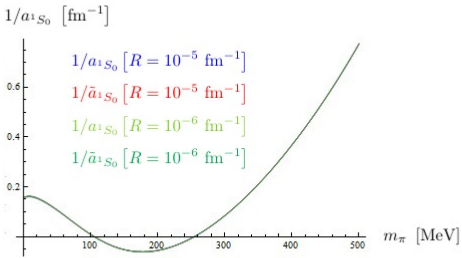
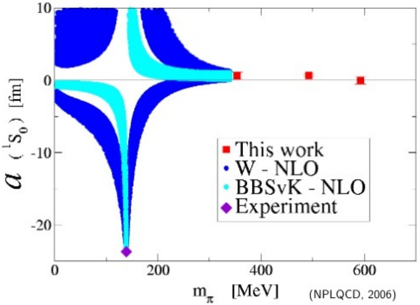
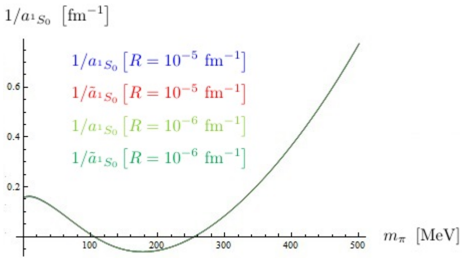


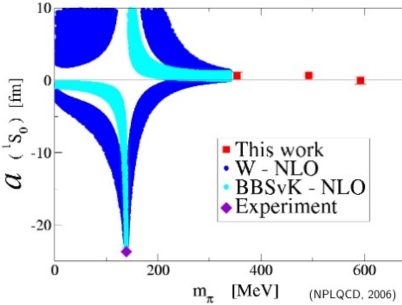



$1/a_{^1S_0}$ [fm^{-1}]
$1/a_{^1S_0}\ [R = 10^{-5}\ \mathrm{fm}^{-1}]$
$1/\tilde{a}_{^1S_0}\ [R = 10^{-5}\ \mathrm{fm}^{-1}]$
$1/a_{^1S_0}\ [R = 10^{-6}\ \mathrm{fm}^{-1}]$
$1/\tilde{a}_{^1S_0}\ [R = 10^{-6}\ \mathrm{fm}^{-1}]$
0.6
0.4
0.2
100
200
300
400
500
m_π [MeV]

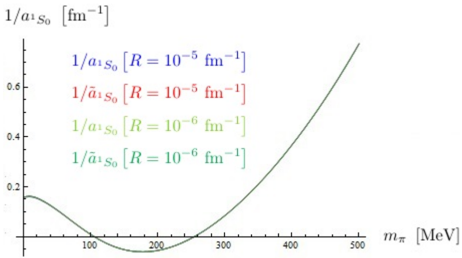

10
0
-10
-20
$a\ (^1S_0)$ [fm]
This work
W - NLO
BBSvK - NLO
Experiment
0
200
400
600
m_π [MeV]

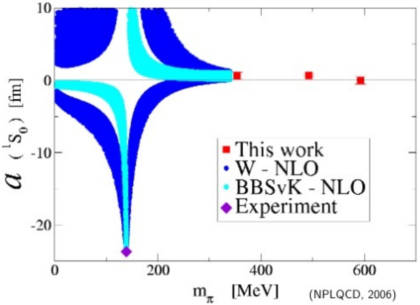
(NPLQCD, 2006)





✓ $$T = T_{\mathrm{LO}}\left[1 + \mathcal{O}\left(\frac{M_{\mathrm{lo}}}{M_{\mathrm{hi}}}, \frac{M_{\mathrm{lo}}}{\Lambda}\right)\right] \Longrightarrow \begin{cases} \text{to minimize CO errors, } \Lambda \gtrsim M_{\mathrm{hi}}; \\ \text{explore } \Lambda \in [M_{\mathrm{hi}}, \infty)\,. \end{cases}$$





(NPLQCD, 2006)

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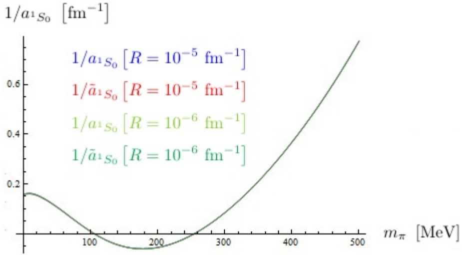
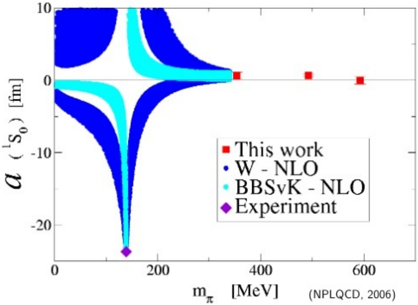
- ✓ Suspected logarithmic residual cutoff dependence: $\sim \frac{\log \Lambda}{\Lambda}$.





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- ✓ Suspected logarithmic residual cutoff dependence: $\sim \frac{\log \Lambda}{\Lambda}$.
- ✓ This could be a good test for our EFT.

Thank you :)