Resonant antihydrogen formation in antiproton-positronium collisions

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Plan

- GBAR (Gravitational Behaviour of Antihydrogen at Rest) project
- Theoretical framework
  1. Merkuriev-Faddeev equations
  2. Problem of the degeneracy of the excited states.
- Results
- Calculations in progress.

The GBAR project

- GBAR: Gravitational Behaviour of Antihydrogen at Rest.
- Experimental project supported by CERN (2012).
- International collaboration of 49 physicits from 14 different institutes.
- Equivalence principle verification : mesure the free fall of ultracold $\bar{H}$ to study the behavior of antihydrogen in a gravitational field.

The GBAR project

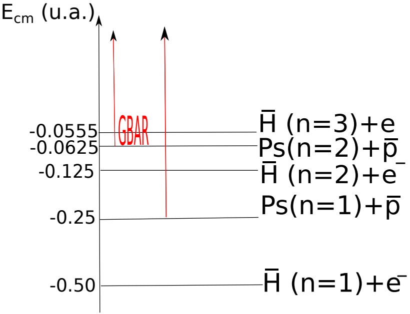
- $\bar{H}^+$ production controled by,

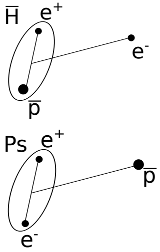
$$\begin{aligned} \bar{p} + Ps &\rightarrow \bar{H} + e^- \\ \bar{H} + Ps &\rightarrow \bar{H}^+ + e^- \end{aligned}$$

- Looking for production beams optimisation.
- Experimental cross sections insufficiently known
  at the GBAR energy levels.
- Theoretical calculations needed. Benchmark contribution of ab-initio methods.

Theoretical framework

$(\bar{p}, e^+, e^-)$ system thresholds.





n principal quantum number.

Theoretical framework

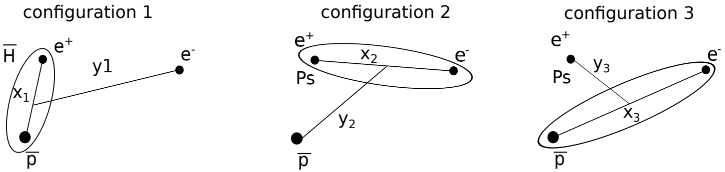
- Hamiltonian considered to describe the 3-body system in the laboratory frame

$$H_{lab} = \frac{\mathbf{p}_{\bar{p}}^2}{2m_{\bar{p}}} + \frac{\mathbf{p}_{e^+}^2}{2m_e} + \frac{\mathbf{p}_{e^-}^2}{2m_e} - \frac{\alpha\hbar c}{|\mathbf{r}_{e^+} - \mathbf{r}_{e^-}|} - \frac{\alpha\hbar c}{|\mathbf{r}_{\bar{p}} - \mathbf{r}_{e^+}|} + \frac{\alpha\hbar c}{|\mathbf{r}_{\bar{p}} - \mathbf{r}_{e^-}|}$$

α fine structure constant.

- We work in the center of mass frame.

$$H = H_{cm} + H^{int}$$



- Intrinsic coordinates defined with the Jacobi coordinates system
- The Hamiltonian can be written for each configuration,

$$H^{int} = H_0(x_i, y_i) + V_1 + V_2 + V_3.$$

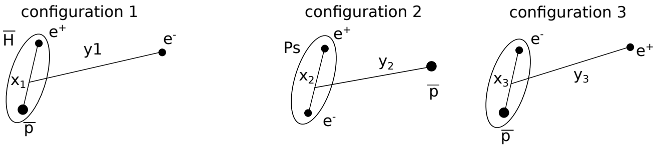
Theoretical framework

- The Merkuriev-Faddeev equations represent a mathematically rigorous ab-initio formulation of the scattering theory for 3-particle systems.
  In contrast to Schrödinger or Lippman-Schwinger equations, which fail to provide an unique solution for the multichannel scattering problem with N>2 particles.

Theoretical framework

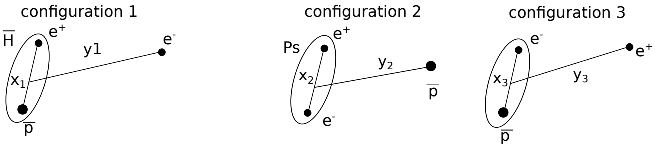
- In this frame work the Wave function is splited in 3 Merkuriev-Faddeev amplitudes,

$$\Psi = F_1 + F_2 + F_3.$$



In the Merkuriev-Faddeev equations, potentials are separated in two parts, a long range part and a short range part,

$$V_i(x_i) = V_i^{(\ell)}(x_i, y_i) + V_i^{(s)}(x_i, y_i).$$



Theoretical framework

Merkuriev-Faddeev equations,

$$\begin{aligned}
(E - H_0 - V_1^{(\ell)} - V_2^{(\ell)} - V_3^{(\ell)})F_1 &= V_1^{(s)}(F_1 + F_2 + F_3) \\
(E - H_0 - V_1^{(\ell)} - V_2^{(\ell)} - V_3^{(\ell)})F_2 &= V_2^{(s)}(F_1 + F_2 + F_3) \\
(E - H_0 - V_1^{(\ell)} - V_2^{(\ell)} - V_3^{(\ell)})F_3 &= V_3^{(s)}(F_1 + F_2 + F_3)
\end{aligned}$$

The Schrödinger equation is the sum of the three equations.

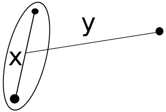
Theoretical framework

- Each Merkuriev-Faddeev amplitude is projected onto a partial wave basis,

$$F_i = \sum_L F_i^L .$$

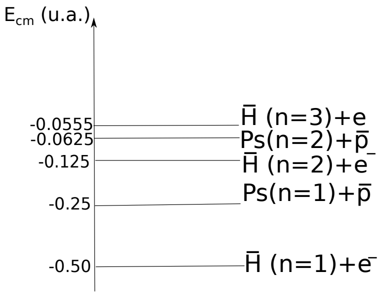
$$\text{and} \quad F_i^L = \sum_{\hat{\ell}_x + \hat{\ell}_y = \hat{L}} F^L_{i,\ell_x \ell_y}$$

- where $\hat{L}$ is the total orbital momentum $\hat{\mathbf{L}} = \hat{\ell}_x + \hat{\ell}_y$



- Within this framework we are able to determine bound states, collision states and to compute cross sections (differential, partial, total).

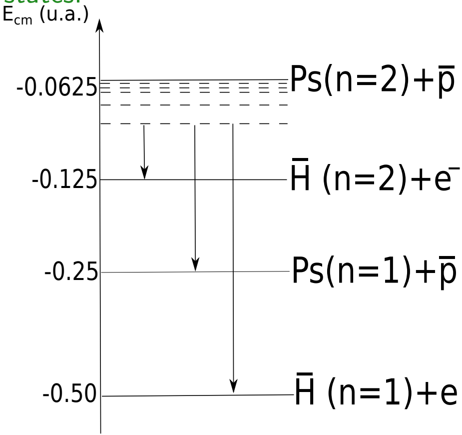
Problem of degeneracy of the excited states

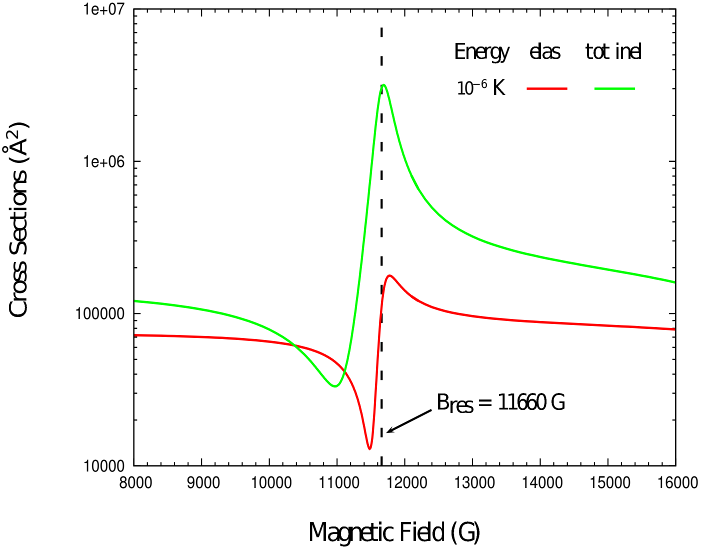


- We focus on the reactions above the $\bar{H}(n=2)$ threshold.
- One of the main technical difficulties comes from the treatment of the degeneracy.

Problem of degeneracy of the excited states

- In the asymptotic region the coupling potential between the degenerate states has an $\frac{1}{y^2}$ behavior.
- The coupling potential generates effective attraction, this enables the formation three-body "long lived" or quasi-bound states.

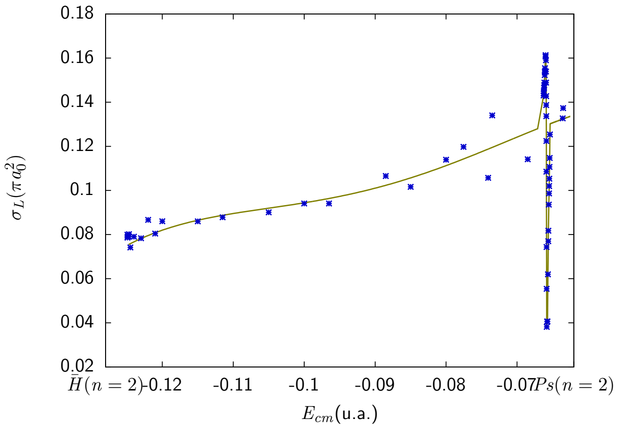




Feshbach resonance found by Jeremy M. Huston *et al* obtaines for an $He + O_2$ collision as a fonction of the magnetic field. Jeremy M. Huston, Musie Beyene and Meykel Leonardo Gonzalez-Martinez, Dramatic reductions in inelastic cross sections for

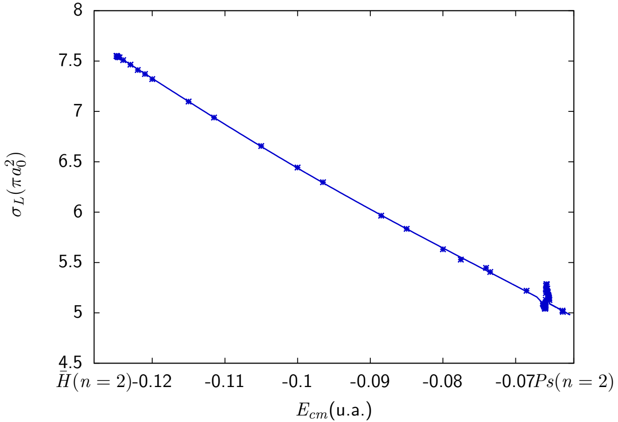
Results

$L = 0$ partial cross section $Ps + \bar{p} \rightarrow \bar{H}^*(2s, 2p) + e^-$



- Partial cross section σ_L in πa_0^2 units (a_0 is the bohr Radius).
- New resonance in the S partial wave cross section.

$L = 0$ partial cross section
$Ps(1s) + \bar{p} \to Ps(1s) + \bar{p}$



- The S wave resonance can be seen in all the reactions.

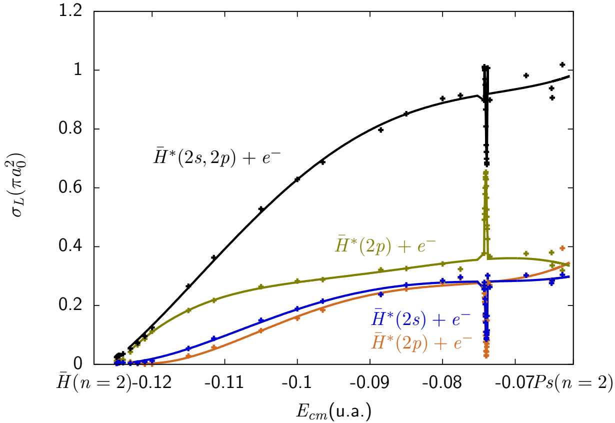
Results

The position of this resonance can be calculated using the complex scaling method ([1]).

Threshold	-Res(E_{res}) a.u.	$\Gamma/2$ a.u.
$H(n=2)$	0.128622631	3.3283[-5]
0.124932	0.1251318	1.82[-6]
$Ps(n=2)$	0.07513977	1.67290[-4]
0.25	0.0658293	8.127[-5]
	0.0633866	2.494[-5]
	0.06274	6.9[-6]

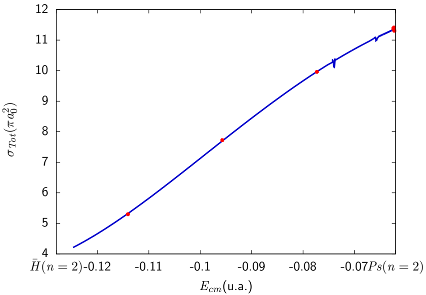
[1] Resonant antihydrogen formation in antiproton–positronium collisions R Lazauskas, P-A Hervieux, M Dufour and M Valdes. J.Phys. B: At. Mol. Opt. Phys. 49(2016)094002

$L = 1$ partial cross section, $Ps + \bar{p} \rightarrow \bar{H}^*(2s, 2p) + e^-$



One new resonance in the P partial wave.

Total cross section $Ps + \bar{p} \rightarrow \bar{H}(1s, 2s, 2p) + e^-$



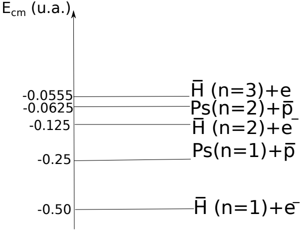
We have calculated the total cross section with a maximum total angular momentum $L_{max} = 7$. The bigger contribution comes from the $L = 2(D)$, $L = 3(F)$ and $L = 4(G)$ partial waves.

The dots in this figure come from the calculations done by A.S. Kadyrov *et al.* [2] with the two-center convergent close-coupling method.

[2] Antihydrogen Formation via Antiproton Scattering with Excited Positronium A. S. Kadyrov, C. M. Rawlins, A. T. Stelbovics, and I. Bray., M. Charlton PHYSICAL REVIEW LETTERS, 183201 (2015)

Calculations in progress.

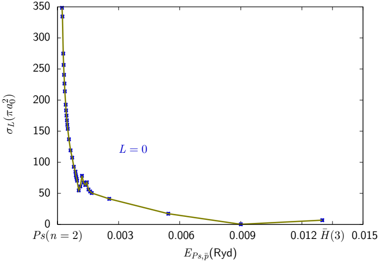
We are now calculating the cross sections above the $Ps(n = 2)$ thresholds.

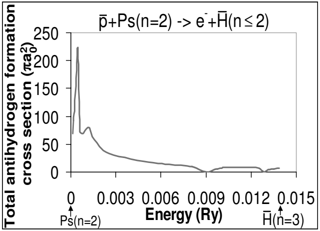


we look forward to find new resonances above the $Ps(n = 2)$ threshold to contribute to the Gbar experiment.

$L = 0$ partial cross sections $Ps(2s, 2p) + \bar{p} \rightarrow \bar{H}(1s, 2s, 2p) + e^-$

Very preliminary results





Chi-yu Hu *et al* [3]

Resonance in the $\bar{H}$ production near the $Ps(n = 2)$ threshold for the First partial wave.

[3] Inducted Long-Range Dipole-Field-Enhanced Antihydrogen Formation in the $\bar{p} + Ps(n = 2) \rightarrow e^- + H(n \leq 2)$ Reaction C.-Y. Hu, D. Caballero Phys. Rev. Lett 88.063401

Thank you for your attention.

The merkuriev separation of the potential.

$$\begin{aligned} V_i^{(s)}(x_i, y_i) &= V_i(x_i) f^{(M)}(x_i, y_i), \\ V_i^{(l)}(x_i, y_i) &= V_i(x_i)(1 - f^{(M)}(x_i, y_i)). \end{aligned}$$

$f^{(M)}$ is a cut-off function which goes to zero when $x_i \quad y_i \to \infty$ and goes to one when $x_i << y_i \to \infty$.

$$f^{(M)}(x, y) = 2(1 + exp\{\frac{(x/x_0)^{\mu}}{y/y_0 + 1}\})^{-1}$$

where $\mu > 2$ and x_0 and y_0 must be of the size of the system.

Comparison with previous results obtained for the S wave.

1) $\bar{H}(n=1)+e^-$ $\sigma_{ij}(E_{cm})$ en πa_0^2

2) $Ps(n=1)+\bar{p}$ partial cross section between i and j

3) $\bar{H}(n=2)+e^-$

E_{cm}	work	σ_{11}	σ_{13}	σ_{12}
-0.115 u.a.	[3]	0.0900	0.001156	0.00572
	[4]	0.0951	0.001004	0.00558
	This work.	0.0964	0.000891	0.00570
-0.10 u.a.	[3]	0.096	0.001514	0.00585
	[4]	0.1010	0.001641	0.00563
	This work.	0.1015	0.001675	0.00574

[3] C.-Y. Hu, Phys. Rev. A 59, 4813 (1999)

[4]Three-potential formalism for the three-body scattering problem with attractive Coulomb interactions (2008) Z. Papp , C-.Y. Hu , Z. T. Hlousek , B. Konya and S. L. Yakovlev.

- Resonant antihydrogen formation in antiproton–positronium collisions R Lazauskas, P-A Hervieux, M Dufour and M Valdes. J.Phys. B: At. Mol. Opt. Phys. 49(2016)094002(4pp)
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