Hydrodynamics in neutron star inner crust

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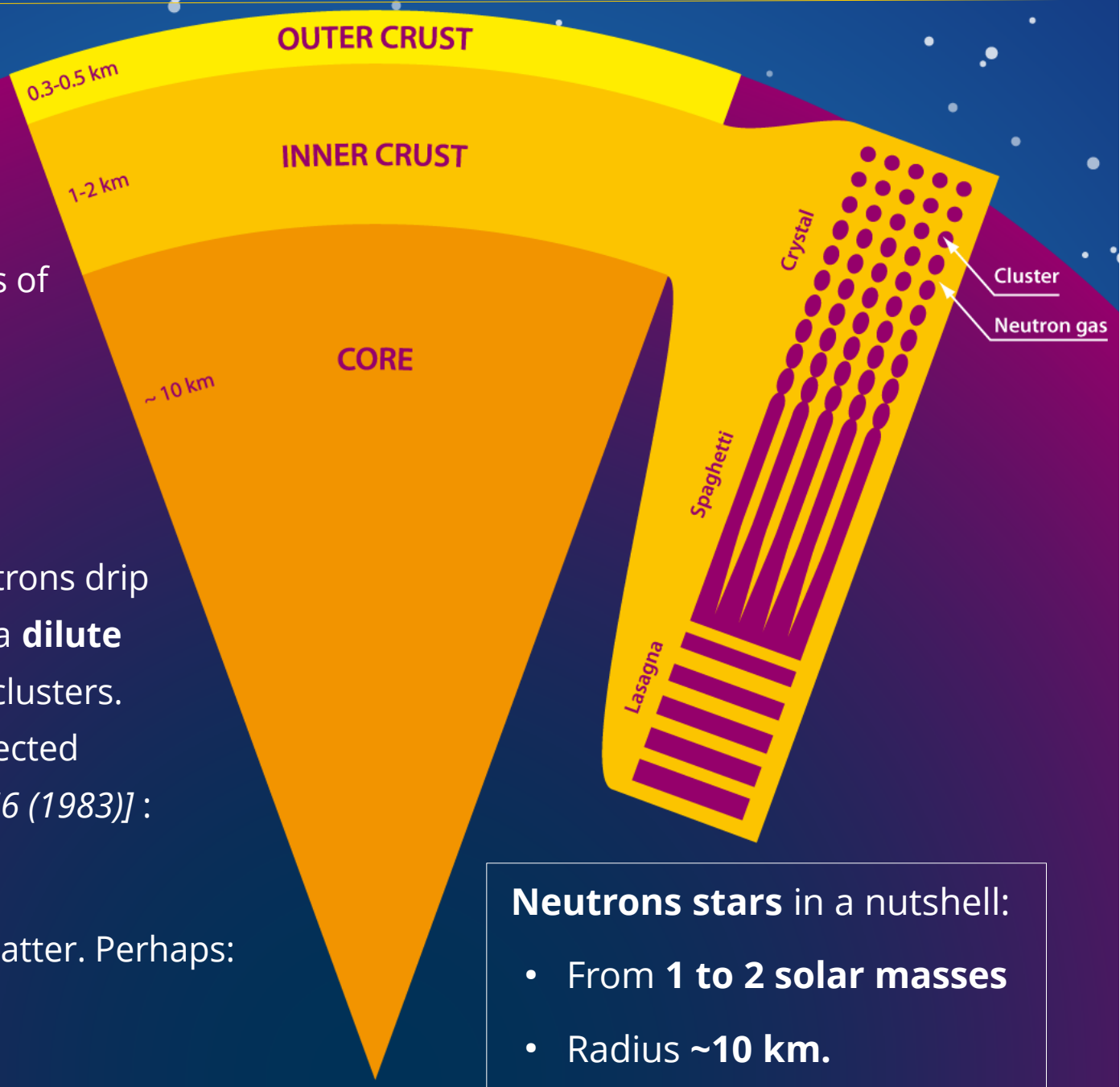
Introduction

Neutrons stars are expected to be organized in **layers:**

- **Outer crust:** nuclei in a gas of relativistic electrons. **Crystalline arrangement** because of the Coulomb interaction.
- **Inner crust:** unbound neutrons drip out of the nuclei and form a **dilute neutron gas** between the clusters. **Different geometries** expected *[Ravenhall et al., PRL 50, 2066 (1983)]* : **crystal**, **rods** and, **plates**.
- **Core:** homogeneous *npe* matter. Perhaps: hyperons, quark matter?



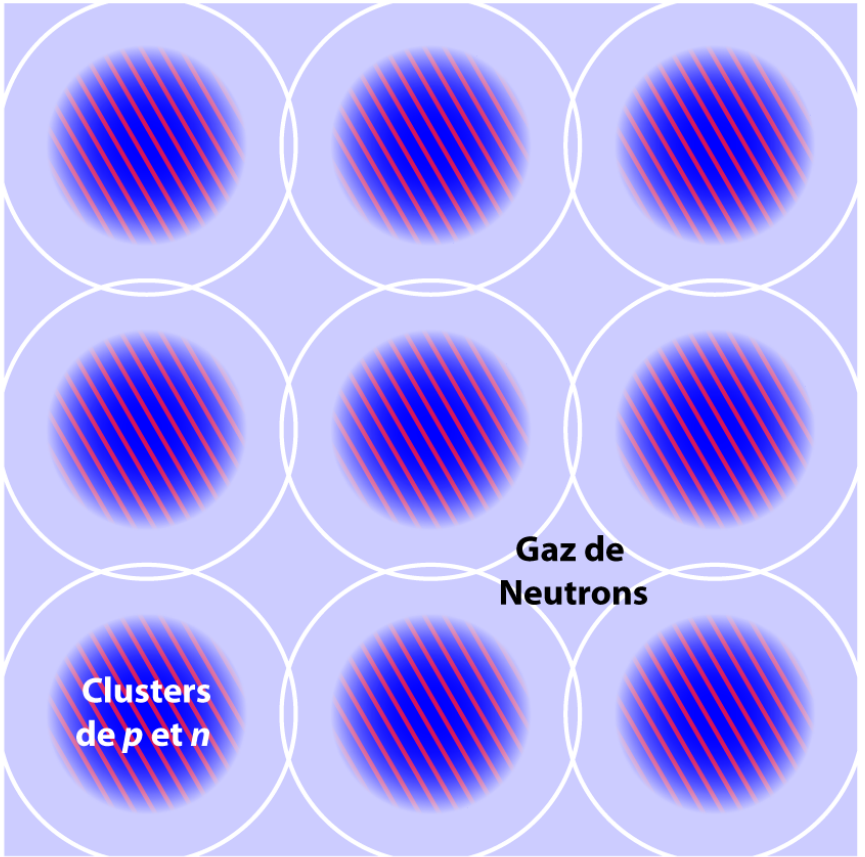
Neutrons stars in a nutshell:

- From **1 to 2 solar masses**
- Radius **~10 km.**

Inner crust description

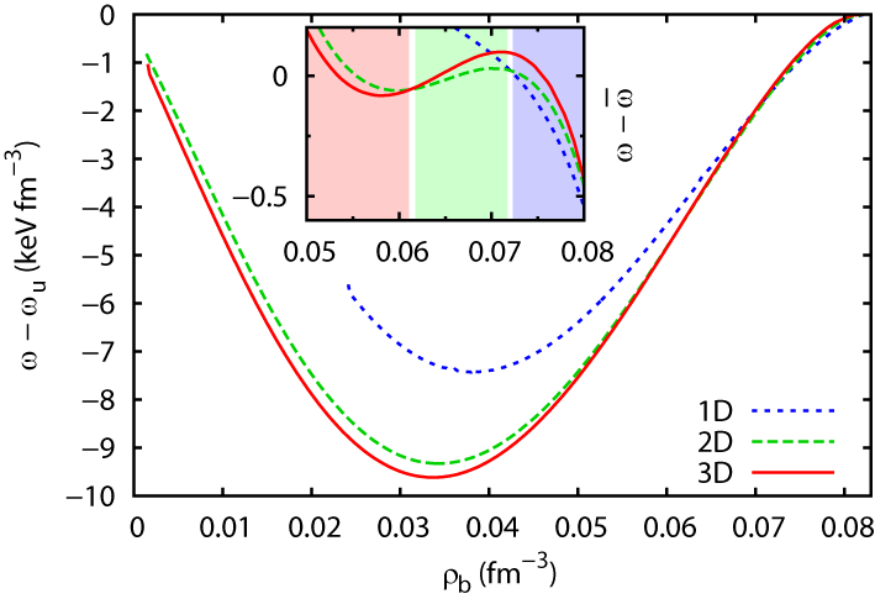
N.M. and M. Urban, Phys. Rev. C **92**, 015803 (2015)

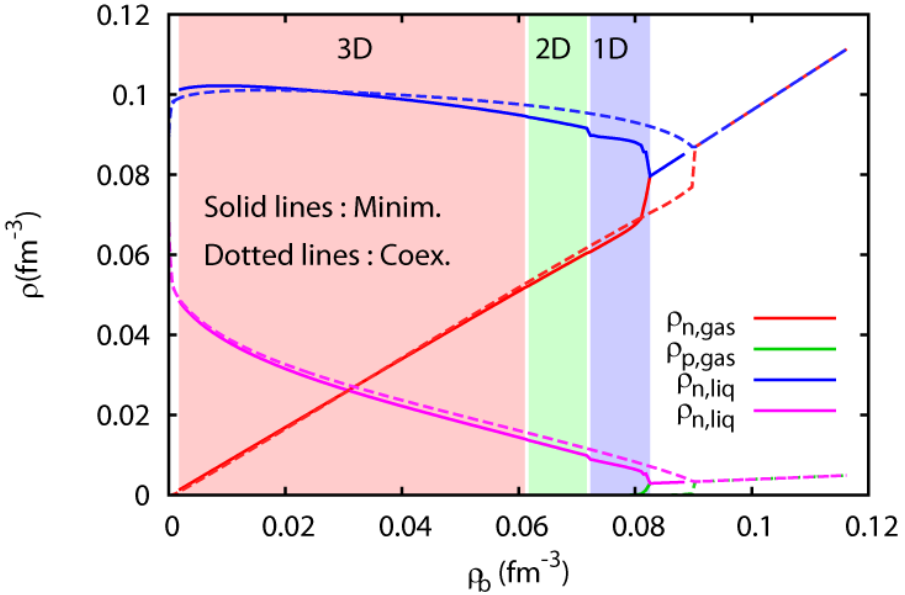
- **Kinetic energy** approximated with **Extended Thomas-Fermi.**
- **Energy minimization** with a **parametrized surface** (diffusivity).
- **Coulomb energy** calculated within **Wigner-Seitz** approximation.



Energy minimization

- **Phase transitions**:
  - **3D** → **2D** ~ 0,06 fm^{-3}
  - **2D** → **1D** ~ 0,07 fm^{-3}
  - **1D** → **Uni.** ~ 0,08 fm^{-3}
- Phase coexistence (dashed line) and **minimization similar results** for the **densities**.
- **Crust-core** transition **earlier** in **ETF** (surface and Coulomb effects).
- **No inverse phase obtained** (Y_p too small because of β-equilibrium).



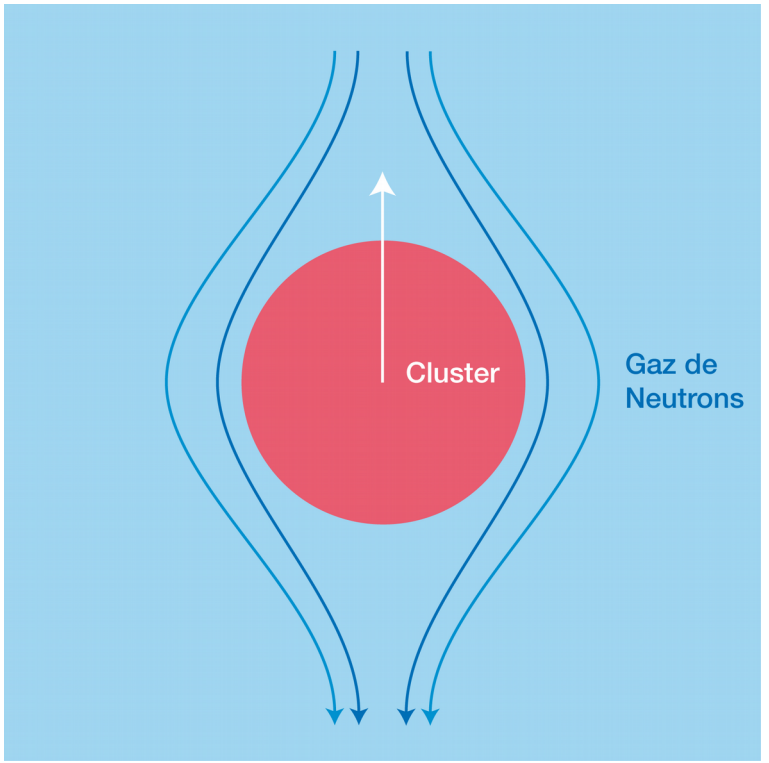


Superfluid hydrodynamics and astrophysical impact

N.M. and M. Urban, arXiv:1606.01126 (e-print)

Entrainment

- **Proton** and **neutron fluids do not move independently** each other, because some neutrons are **bound to the cluster.**
- We talk about **“bound” and “conduction”** neutrons, or in terms of cluster **“effective mass”**.
- Dramatic effect on lattice **phonons, heat capacity**, etc…



Band theory *(solid state physics) [N. Chamel]*

- **Clusters entrain** neutrons with them $\mathbf{A_{eff} > A.}$
- **Superfluidity not included.**

Superfluid hydrodynamics *[A. Sedrakian, P. Magierski and A.Bulgac]*

- **Neutrons** go **through** clusters $\mathbf{A_{eff} < A.}$
- Considered as an **isolated nucleus** inside an **infinite gas.**

Superfluid hydrodynamics

- **Superfluid hydrodynamics** is valid for low-energy excitations (cf. QRPA) :

$$\boldsymbol{v}_n(\boldsymbol{r}) = \frac{1}{2m}\nabla\phi(\boldsymbol{r}) \qquad \phi = \text{phase of } \Delta_n$$

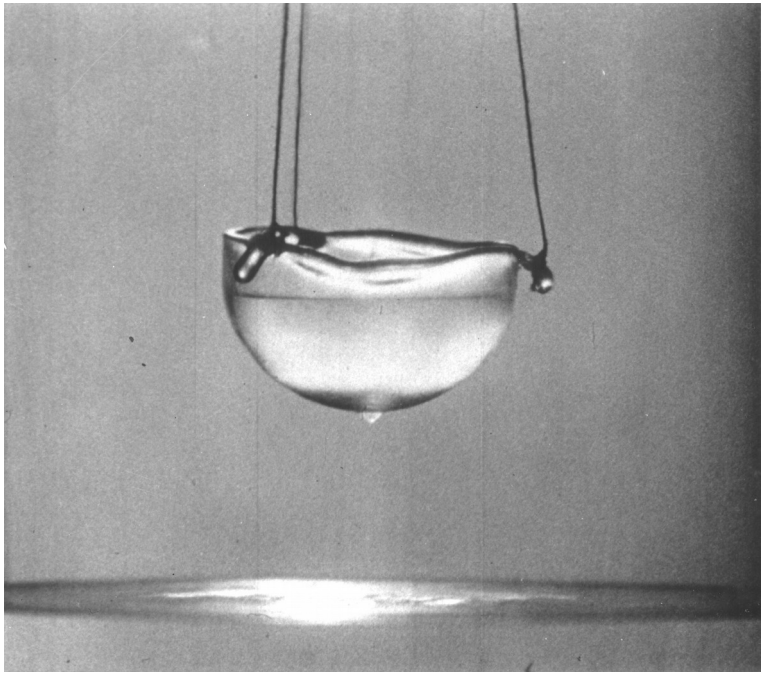
- Ground state: **hydrostatic equilibrium**
  - Clusters with **sharp surface** (**phase coexistence**),
  - Needs **boundary conditions at liquid-gas** interface,

$$\phi^{\text{in}} = \phi^{\text{out}}$$

$$\rho_n^{\text{in}} v_{n\perp}^{\text{in}} - \rho_n^{\text{out}} v_{n\perp}^{\text{out}} = (\rho_n^{\text{in}} - \rho_n^{\text{out}}) v_{p\perp}$$

  - **Periodic lattice,**

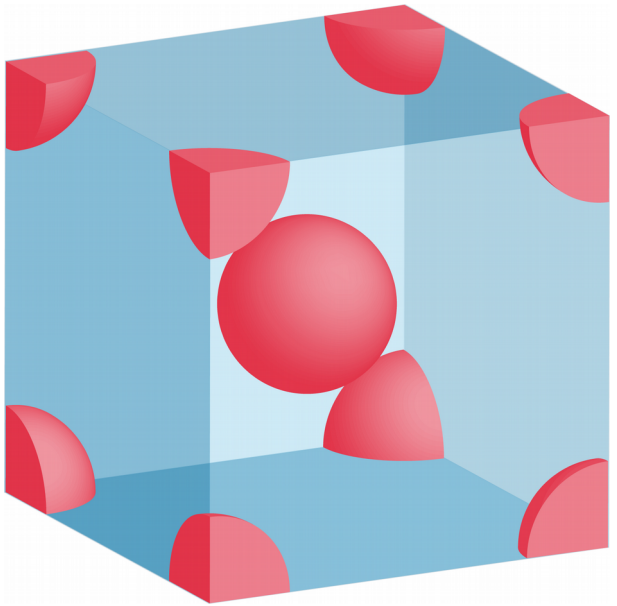
$$\phi(\boldsymbol{r}) = \phi(\boldsymbol{r}+\boldsymbol{L})$$



3D and 2D geometries

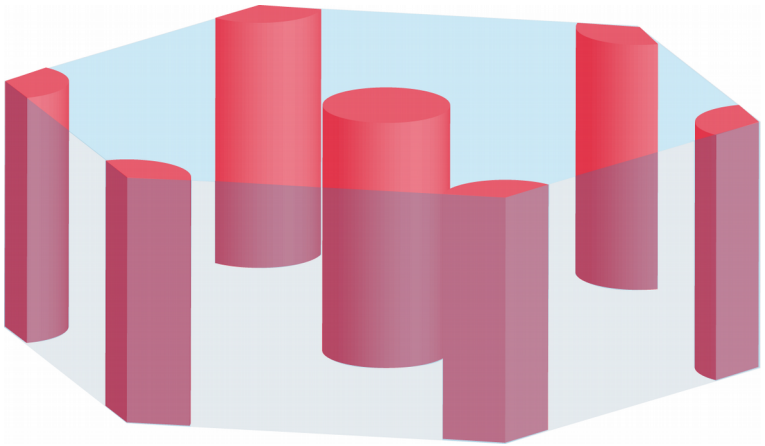
Crystalline phase (3D)

- Inner crust
- **Body Centered Cubic** (BCC) cell
- **Two clusters** per unit cell



« Spaghetti » phase (2D)

- Inner crust
- **Hexagonal** cell
- **One cluster** per unit cell



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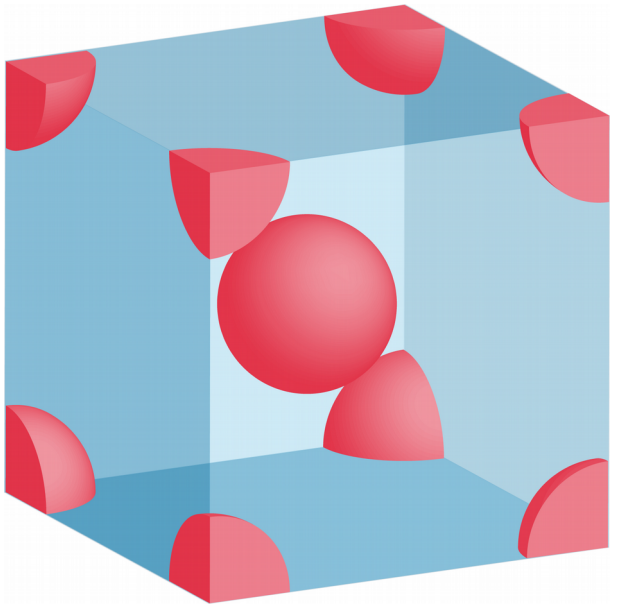

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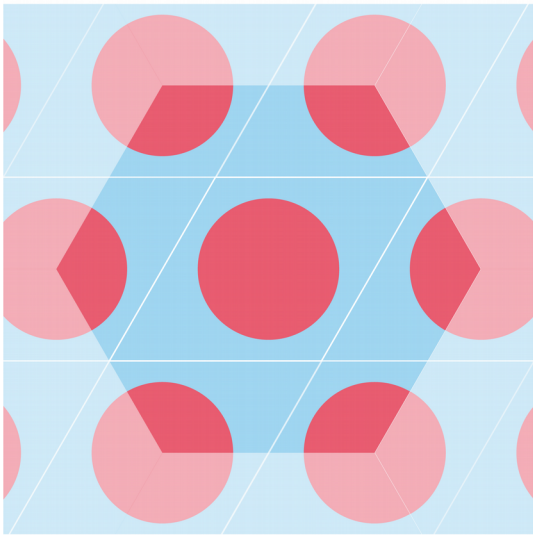


Numerical methods

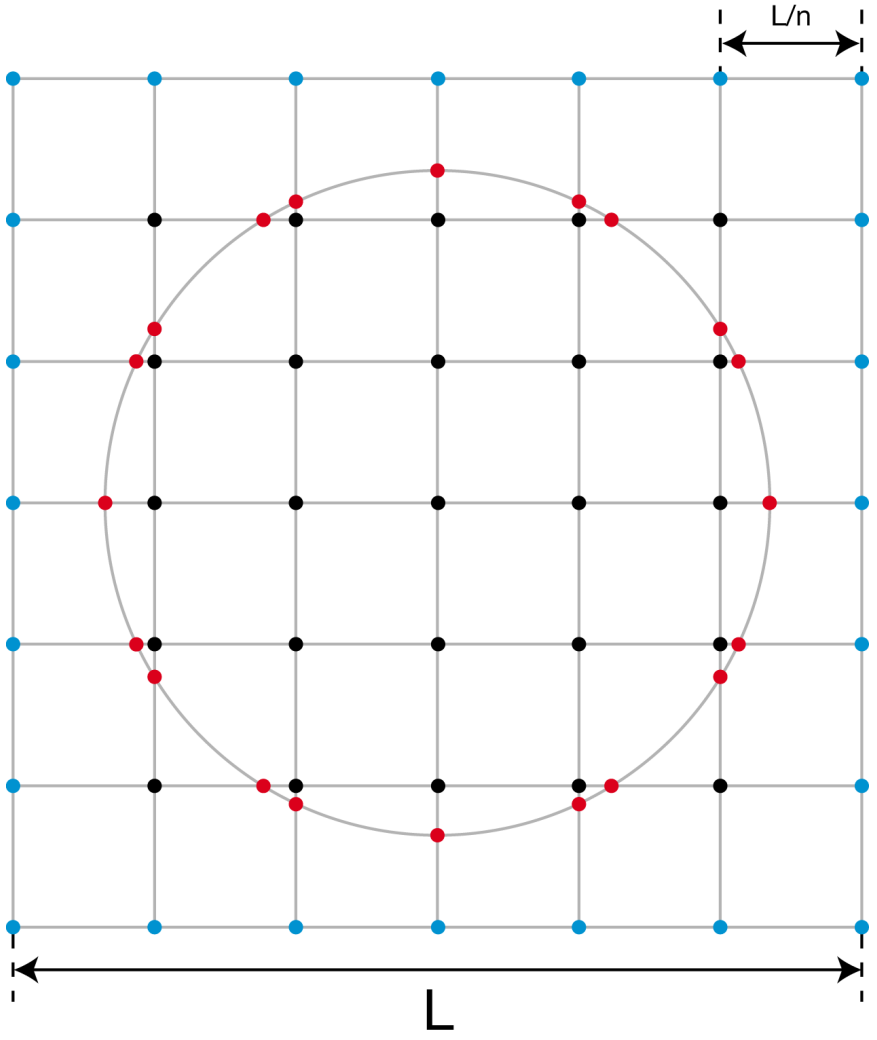
Equations to solve

$$\Delta\phi = 0,$$

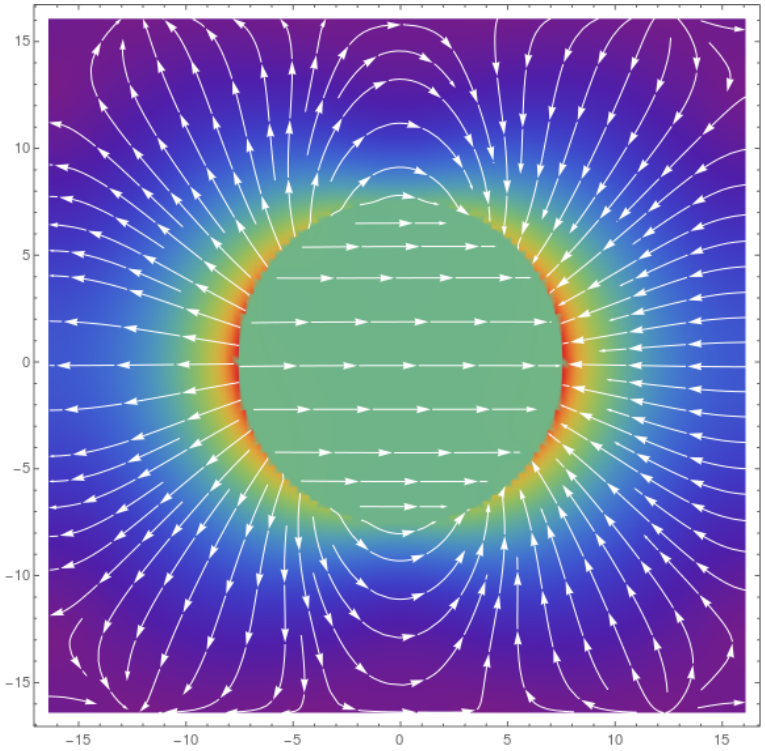
$$\rho_{in}\left(\left.\frac{\partial\phi_{in}}{\partial r}\right|_{r=R} - \vec{v}\cdot\vec{n}\right) = \rho_{out}\left(\left.\frac{\partial\phi_{out}}{\partial r}\right|_{r=R} - \vec{v}\cdot\vec{n}\right),$$

$$\phi(x,y,z) = \phi(x+L,y,z) \quad \text{(analogous for y,z)}$$

- **We discretize equations,**
  - Regular **mesh**,
  - **Surface** points (lying on grid axis),
  - **Periodicity.**
- Solving a huge **linear system of equations** (~600 GB) and **"sparse matrix"** storage.
- Solved with PARDISO (MKL Intel) until ~160^3 points (done at C.C. IN2P3, Lyon).



Crystalline phase – Streamlines



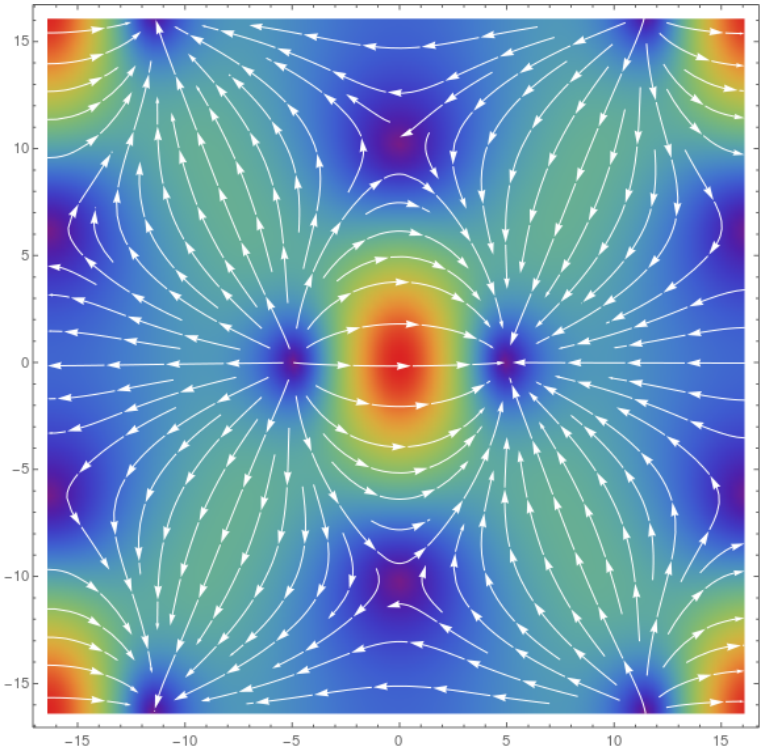
Properties

$R_{CLUSTER}$ = 7.54 fm

L_{CELL} = 38.1fm

ρ_{iN} = 0.0973 fm^{-3}

ρ_{OUT} = 0.0412 fm^{-3}



z = 0 fm

Cluster center

Colors → Norm of *v*
White → Streamlines

z = 8,6 fm

Between clusters

Crystalline phase – Potential Φ

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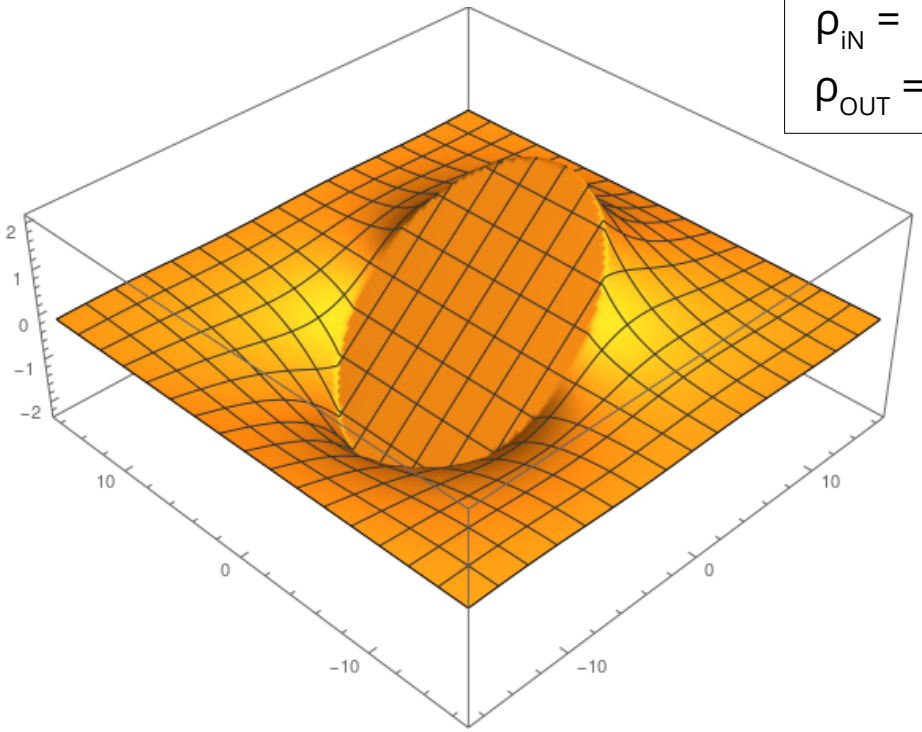

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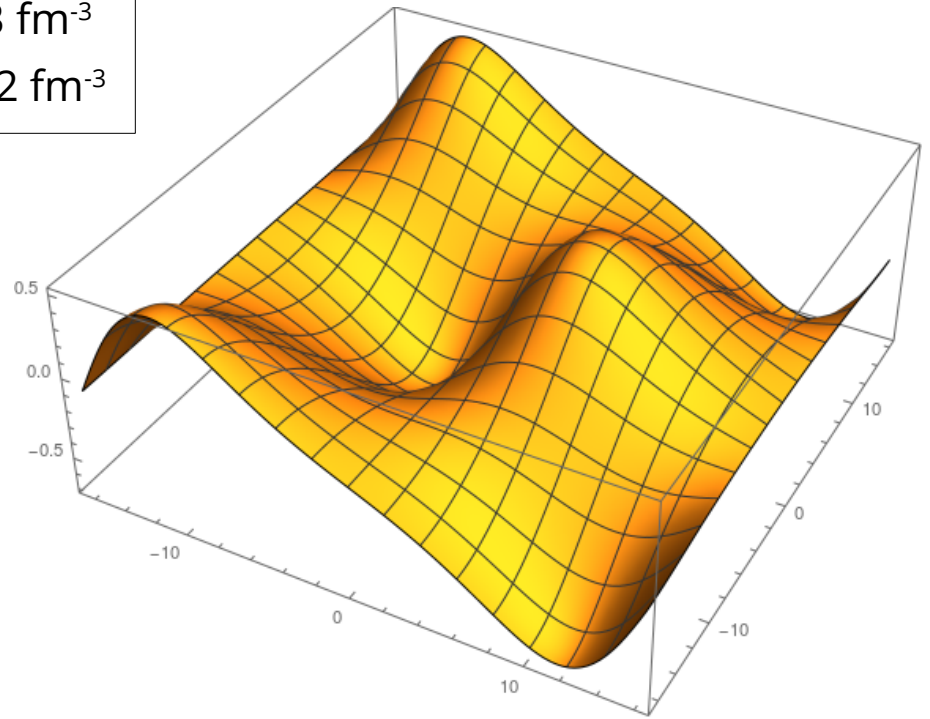


z = 8,6 fm

Between clusters

Cluster properties obtained from ETF calculation

“Spaghetti” phase – Streamlines

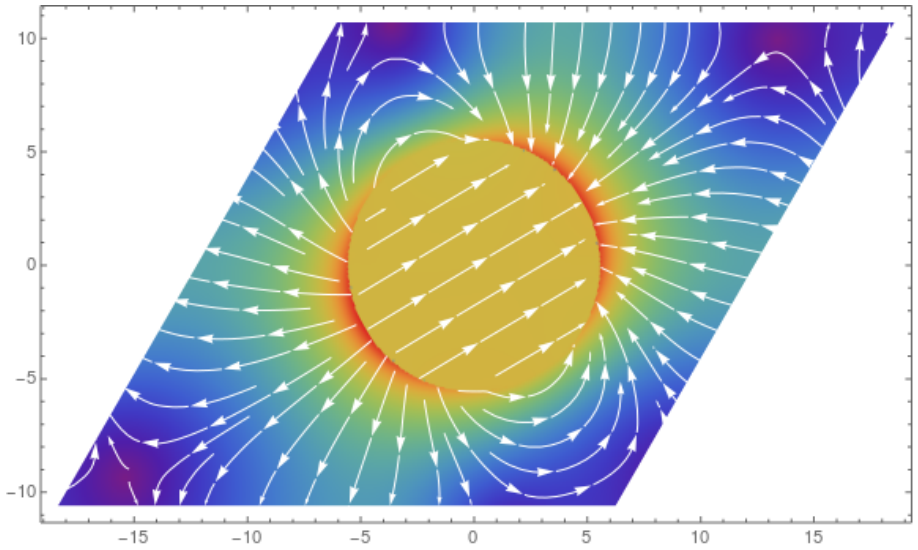
Properties

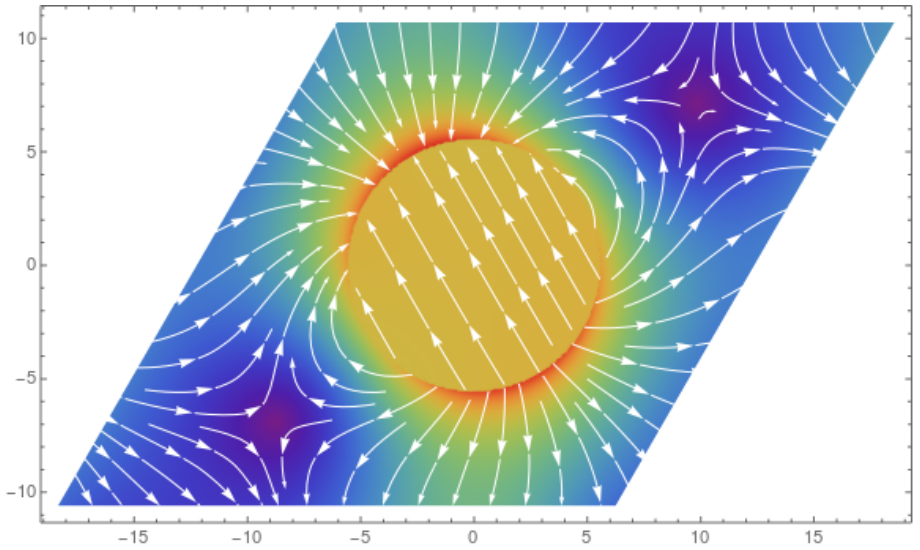
$R_{CLUSTER}$ = 5.53 fm

L_{CELL} = 24.7 fm

ρ_{iN} = 0.0943 fm^{-3}

ρ_{OUT} = 0.0528 fm^{-3}





$\mathbf{v_p} = v_p \mathbf{e_1}$

Colors → Norm of *v*
White → Streamlines

$\mathbf{v_p} = v_p \mathbf{e_2}$

“Spaghetti” phase – Potential Φ

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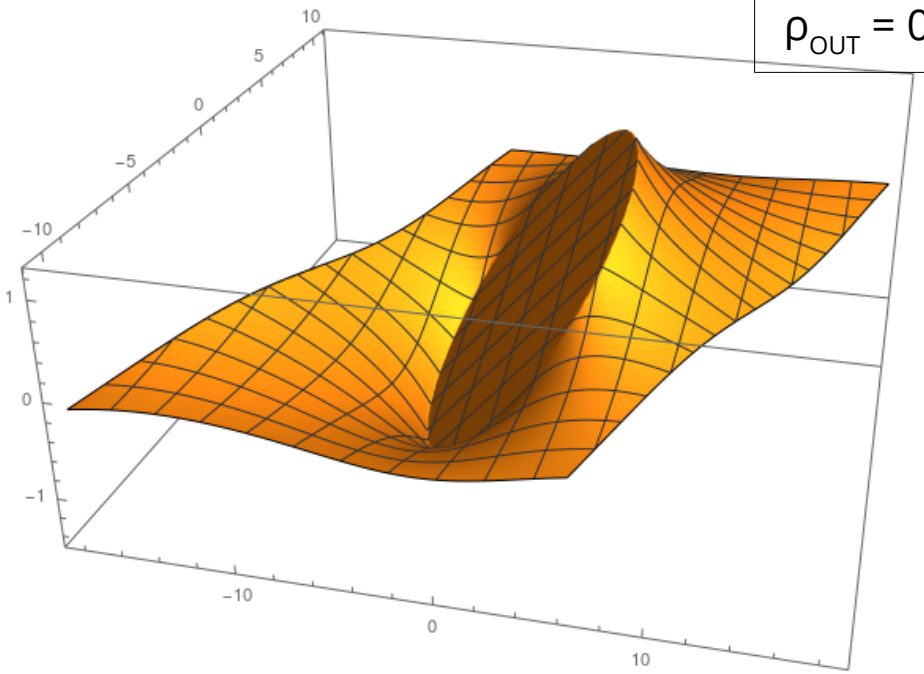
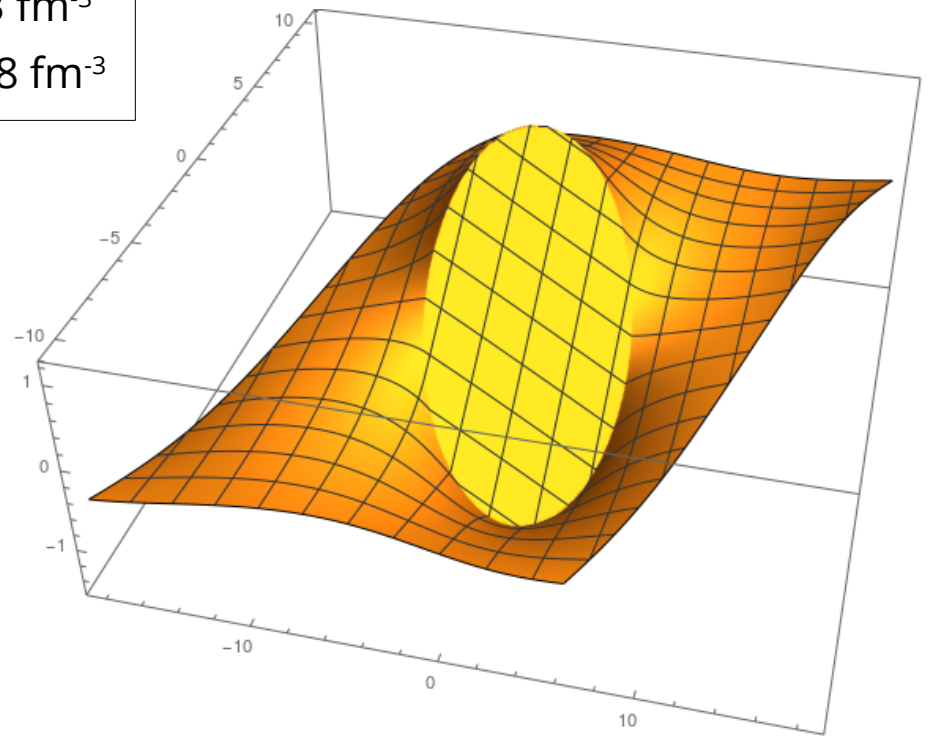


$\mathbf{v_p} = v_p \mathbf{e_1}$

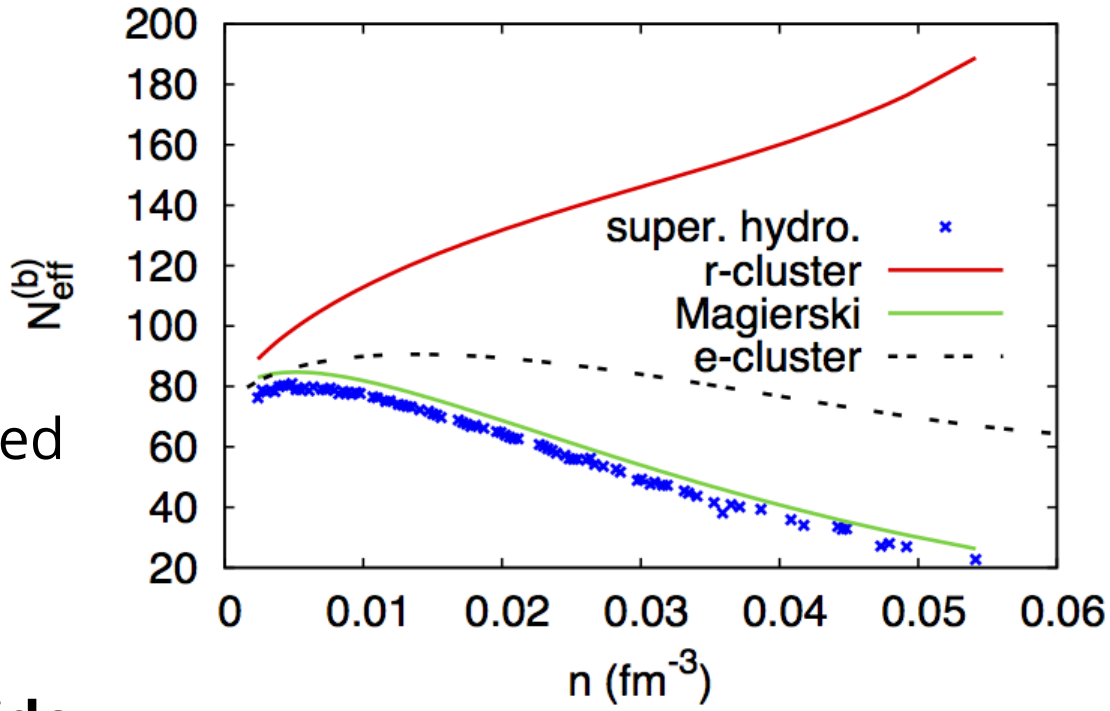


$\mathbf{v_p} = v_p \mathbf{e_2}$

Cluster properties obtained from ETF calculation

“Bound” neutrons

- **r-cluster**: number of neutrons in the cluster.
- **Magierski-Bulgac**: analytical solution for a **cluster in an infinite gas** *[NPA 738 (2004) 143]*
- **Numerical:** the present calculations.
- **e-cluster:** neutron kind defined with respect to their energy.

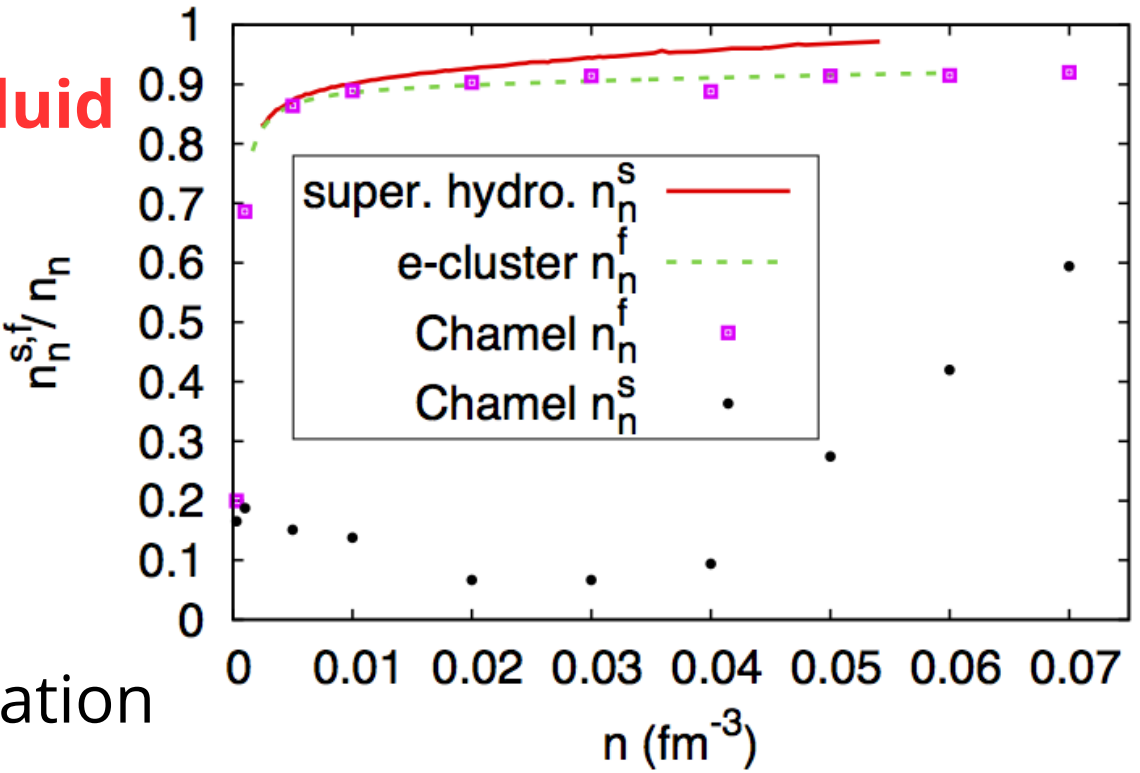


Comments

- The three **approaches coincide** at low densities.
- **Periodicity has a small effect**.

Superfluid neutrons

- **Superfluid density,** obtained with **superfluid hydrodynamics.**
- Results larger than Chamel from **band structure.**
- The **e-cluster results** agrees with the calculation of **n^f of Chamel.**
- Here, one observes that **$n^s > n^f$**. *Which impact on astrophysics?*



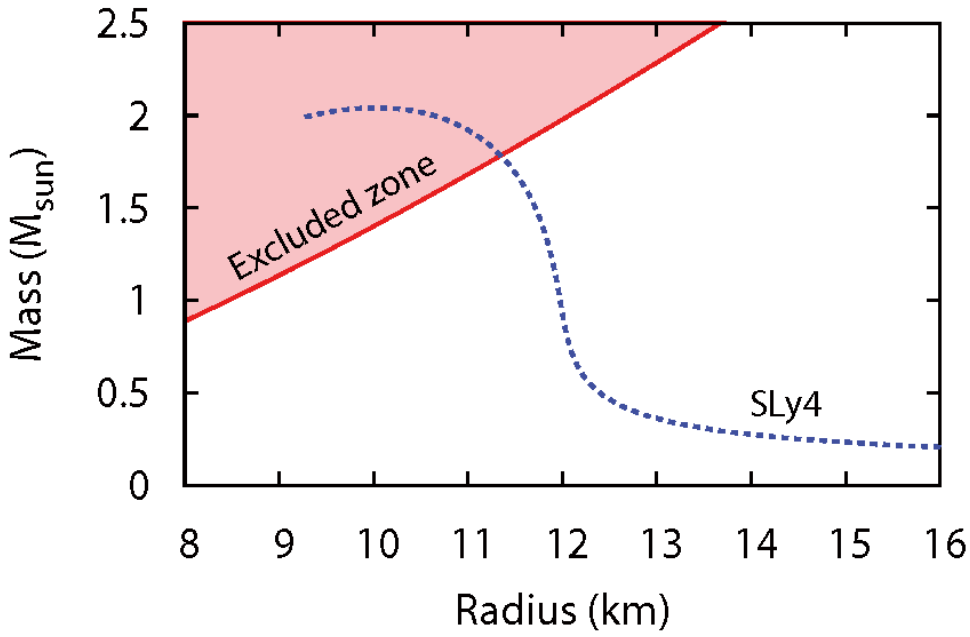
Possible reason for discrepancy?

- In condensed matter physics:
  band gap >> pairing gap
- Neutron star crust:
  band gap < pairing gap

Astrophysical impact – Glitches

Glitch: sudden increase of the rotation velocity

- **Transfer of angular momentum** from superfluid to rigid parts of the star.
- **Strongly affected by the superfluid properties.**
- Calculated with: **superfluid** properties (**entrainment**) and **clusters.**



- **Constraint on possible maximal mass and radius for *Vela*:**
  - **Super. Hydro.: ~1.8 $M_\odot$**
  - Band theory: ~ 0.7 $M_\odot$ [Chamel, PRL 110, 011101 (2013)]

Better agreement of our constraint on the maximal mass and radius with the **glitch activity observations.**

Summary

- **Inner crust description** done with **ETF approximation**.

- **Superfluid hydrodynamics** description of neutron **entrainment** within **2D and 3D geometries.**
  → **1D calculations** (analytical) already done, for impermeable surface *[M. Urban, M. Oertel and L. Di-Gallo]*

- Effect of **periodicity in the lattice.**

- Impact on the **astrophysical properties** of the neutron star.

Outlook

- **Phonon velocity.**
- Which **impact** on **neutron star cooling ?**
- Adding **surface tension** and **neutron skin** to the hydrodynamics.
- **Time dependent** hydrodynamic model: **density waves.**