Stochastic mean-field approach to fission observables

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Theories for nuclear fission

- **Conventional strategy**
  1. select a set of relevant collective degrees of freedom $Q$
     ex. elongation, mass asymmetry, etc.
  2. construct potential energy $V(Q)$ and inertial parameters
  3. solve the equations of motion of $Q$ to get fission observables
     lifetime, fragment mass/charge distribution, etc.

- **Macroscopic** approaches based on the liquid-drop model have been successful
- Fully **microscopic** theories for fission are still under development
- With microscopic approaches ...
  - energy density functional (EDF) theory is employed
  - only EDF and initial conditions are needed as input

We aim to establish a microscopic theory for description of the fission process
Microscopic approaches to fission

Time-Dependent Hartree Fock (TDHF)

Compared to TDGCM,

- No need to select collective coordinates
- No adiabaticity is assumed
- Collective motion is nearly classical
- No spontaneous symmetry breaking
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Negele et al, PRC17, 1098 (1978),
Umar and Simenel, PRC89, 031601 (2014).
Goutte et al PRC71, 024316 (2005),
Regnier, Dubray, Schunk, and Verrière, PRC93, 054611 (2016).
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Beyond-mean-field effect must be taken into account

\[
i\hbar \frac{\partial}{\partial t} \rho(t) = [h[\rho(t)], \rho(t)]
\]

e.g., fragment mass distribution

\[\text{yield} \quad A_1 \quad A_2\]

\[\text{fragment mass}\]
Our method:

**Stochastic mean field (SMF) theory**

- **Evolution of a quantum wavepacket is simulated by an ensemble of TDHF trajectories**
  
- Quantum fluctuation at $t = 0$ is taken into account by random sampling of one-body density matrix $\{\rho^{(n)}\}$

\[
\rho^{(n)}(t = 0) = \rho^{(n)}(t = 0) + \delta\rho^{(n)}
\]

- Gaussian distribution is assumed for $\delta\rho^{(n)}$:

\[
\begin{align*}
\delta\rho^{(n)}_{i,j} &= 0 \\
\delta\rho^{(n)}_{i,j} \delta\rho^{(n)*}_{i',j'} &= \frac{1}{2} \delta_{ii'} \delta_{jj'} [n_i(1-n_j) + n_j(1-n_i)]
\end{align*}
\]

- $n_i$: occupation number

- Evolution of a quantum wave packet is simulated by an ensemble of classical (TDHF) trajectories

\[
i\hbar \frac{\partial}{\partial t} \rho^{(n)} = [h[\rho^{(n)}], \rho^{(n)}]
\]
Application to spontaneous fission of $^{258}\text{Fm}$

- interaction: SLy4d (+ pure pairing force)
- 338 events are generated

\[
\rho_{ij}^{(n)}(t = 0) = \delta_{ij} n_i + \delta\rho_{ij}^{(n)} =
\]

\[
\begin{pmatrix}
1 & 1 \\
\delta\rho_{hp} & 0 & 0 \\
\end{pmatrix}
\]
TDHF starting from $Q = 160\, \text{b}$
Total kinetic energy of fragments

- Width of distribution is reasonably reproduced
- Peak position is shifted from TDHF value
- Does not reproduce the asymmetric shape of TKE distribution

\[
\begin{align*}
\overline{\text{TKE}}(\text{Expt.}) &= 215.5 \text{ (MeV)} \\
\overline{\text{TKE}}(\text{SMF}) &= 202.4 \text{ (MeV)} \\
\overline{\text{TKE}}(\text{TDHF}) &= 240.7 \text{ (MeV)} \\
\sigma_{\text{TKE}}(\text{Expt.}) &= 19.3 \text{ (MeV)} \\
\sigma_{\text{TKE}}(\text{SMF}) &= 28.4 \text{ (MeV)}
\end{align*}
\]

Hulet et al., PRL\textbf{56}, 313 (1986)
Fragment-mass distribution

Asymmetric configurations partially reproduced by SMF

\[
\sigma_A (\text{Expt.}) = 10.6 \\
\sigma_A (\text{SMF}) = 6.1 \\
\sigma_A (\text{TDHF}) = 2.4
\]
Works ongoing:
Sensitivity of results to initial condition?
1. Truncation in s.p. space
2. Starting point
Works ongoing:

1. Fit $\Delta \varepsilon$ to reproduce $E^*$ at given $Q_2$.

2. Start from $Q_2 < Q_2^{th}$.

$E^* = E[\rho^{(n)}] - E[\rho^{(n)}(t = 0)]$
Works ongoing:

1. Fit $\Delta \epsilon$ to reproduce $E^*$ at given $Q_2$
2. Start from $Q_2 < Q_2^{th}$

$$E^* = E[\rho^{(n)}] - E[\rho^{(n)}(t = 0)]$$
Summary

• **Aim:** Fully microscopic and dynamical description for fission

• **We tested the SMF theory to take into account the quantum fluctuations missing in TDHF**
  – Fluctuation of $\rho_{ij}$ is introduced at $t = 0$ by random sampling
  – Possible to obtain realistic TKE and fragment-mass distributions

• **Spontaneous fission of $^{258}$Fm**
  – Fluctuations in observables are improved compared to TDHF

• **Sensitivity to Initial condition will be further studied**
Application to spontaneous fission of $^{258}\text{Fm}$

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- 338 events are generated

$$E^* = E[\rho^{(n)}] - E[\rho^{(n)}(t = 0)]$$

$$\rho_{ij}^{(n)}(t = 0) = \delta_{ij}n_i + \delta \rho_{ij}^{(n)} =$$
Microscopic dynamical approaches

Time-Dependent Hartree Fock (TDHF)
- Umar and Simenel, PRC 89, 031601 (2014).

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Time-dependent generator-coordinate method (TDGCM)
- Regnier, Dubray, Schunk, and Verrière, PRC 93, 054611 (2016).

- Quantum treatment of collective degrees of freedom
- Nontrivial selection of collective coordinates
- Numerical cost rises rapidly with number of coordinates